Program - *Workshops*

Consider attending one of the workshops and take advantage of the LOW registration fee. Registration will be available online. **Subject to cancellation if the minimum number of registrations is not achieved. Must register by April 22, 2019.

WORKSHOP 1 – Physics-Based Modeling of Gas Turbine Secondary Air Systems

Sunday, June 16
8:00 am – 5:00 pm
Cost: $300 per person

In gas turbines used for power generation and aircraft propulsion, the main flow paths of compressors and turbines are responsible for the direct energy conversion. To ensure acceptable life (durability) under creep, LCF, and HCF from operational transients causing high temperatures and their gradients in critical engine components, around 20% of the compressor air flow is used for cooling and sealing. This is analogous to blood, water, and air flow within a human body for its proper functioning. The main thrust of this workshop is to develop a clear understanding of the underlying flow and heat transfer physics and the mathematical modeling of various components of gas turbine secondary air systems (SAS). In addition to developing a clear understanding of the key concepts of thermofluids, the workshop will discuss vortex, windage and disk pumping in rotor/stator cavities, centrifugally-driven buoyant convection in compressor rotor cavities, pre-swirler systems, multiple reference frames, hot gas ingestion and rim sealing, and whole engine modeling (WEM) using nonlinear multisurface forced vortex convection links with windage in a layered approach. Additionally, the workshop will provide a design-friendly overview of rotating compressible flow network methodology along with robust solution techniques, physics-based post-processing of 3-D

CFD results, and the generation of entropy map for design optimization. A number of design-relevant examples will also be presented in the workshop.

Notes: Five complimentary, autographed copies of Gas Turbines: Internal Flow Systems Modeling (Cambridge Aerospace Series) will be distributed among workshop attendees using a random draw.

Learning Objectives

- Develop a strong foundation in flow and heat transfer physics of various components of gas turbine secondary air systems
- Develop an intuitive understanding of 1-D compressible duct flows under the coupled effects of area change, friction, heat transfer, and rotation
- Gain knowledge in developing accurate physics-based and solution-robust secondary air flow network models
- Gain knowledge in detecting input and modeling errors in their flow network models
- Interpret results from their models for design applications.
- Develop skills to hand-calculate results to perform sanity-checks of predictions by design tools as well as to validate these tools during their development and continuous improvement
- Improve your engineering productivity with reduced design cycle time

Outline

8:00 – 10:00 am
Module 1: An Overview of Secondary Air Systems

- Role of Secondary Air Systems (SAS) modeling in gas turbine design engineering
- The concept of physic-based modeling
- Key components of SAS
- Flow network modeling and robust solution techniques
- Role of 3-D CFD in SAS modeling
- Physics-based post-processing of CFD results
- Entropy map generation and application

10:00 – 10:15 am Coffee Break

10:15 am – 12:00 pm

Module 2: Special Concepts of Secondary Air Systems – Part I

- Free vortex
- Forced vortex
- Rankine vortex
- Windage
- Compressible flow functions
- Loss coefficient and discharge coefficient for an incompressible flow
- Loss coefficient and discharge coefficient for a compressible flow

12:00 pm – 1:00 pm Group Lunch

1:00 – 2:00 pm

Module 3: Special Concepts of Secondary Air Systems – Part II

- Euler's turbomachinery equation
- Rothalpy
- Multiple reference frames
- Pre-Swirler system
- Rotor disk pumping

2:00 – 3:00 pm

Module 4: Physics-Based Modeling – Part I

- Stationary and rotating orifices and channels
- Rotor-stator and rotor-rotor cavities
- Windage and swirl distribution
- Centrifugally-driven buoyant convection in compressor rotor cavity with and without bore flow

3:15 – 4:00 pm

Module 5: Physics-Based Modeling – Part II

- Hot gas ingestion
- Turbine rim sealing
- Coupling with rotor-stator cavity purge flow and windage

4:00 – 5:00 pm

Module 6: Physics-Based Modeling – Part III

- Whole engine modeling (WEM)
- Multisurface forced vortex convection link with windage
- Junction treatment in the network of convection links
- Layered flow network modeling methodology
- Key recommendations on SAS modeling

Earn 7 Professional Development Hours (PDH's) and receive a certificate of completion!

Instructor:

Dr. Bijay (BJ) K. Sultanian, PhD, PE, MBA, ASME Life Fellow

Dr. Bijay Sultanian is an international authority in gas turbine heat transfer, secondary air systems, and Computational Fluid Dynamics (CFD). Dr. Sultanian is Founder & Managing Member of Takaniki Communications, LLC, a provider of high-impact, web-based, and live technical training programs for corporate engineering teams. Dr. Sultanian is also an Adjunct Professor at the University of Central Florida, where he has been teaching graduate-level courses in Turbomachinery and Fluid Mechanics since 2006. During his 30+ years in the gas turbine industry, Dr. Sultanian has worked in and led technical teams at a number of organizations, including Allison Gas Turbines (now Rolls-Royce), GE Aircraft Engines (now GE Aviation), GE Power Generation (now GE Power & Water), and Siemens Energy (now Siemens Power & Gas). He has developed several physics-based improvements to legacy heat transfer and fluid systems design methods, including new tools to analyze critical high-temperature components with and without rotation.

During 1971-81, Dr. Sultanian made landmark contributions toward the design and development of India's first liquid rocket engine for a surface-to-air missile (Prithvi) and the first numerical heat transfer model of steel ingots for optimal operations of soaking pits in India's steel plants.

Dr. Sultanian is a Life Fellow of the American Society of Mechanical Engineers, a registered Professional Engineer in the State of Ohio, a GE-certified Six Sigma Green Belt, and an Emeritus Member of Sigma Xi, The Scientific Research Society. He is the author of three graduate-level textbooks: ***Fluid Mechanics: An Intermediate Approach***, published in 2015; ***Gas Turbines: Internal Flow Systems Modeling (Cambridge Aerospace Series)***, published in 2018; and ***Logan's Turbomachinery: Flowpath Design and Performance*** Fundamentals, to be published in 2019.

For the ASME Turbo Expo 2019, he is the Heat Transfer Committee Point Contact, a role he also had for Turbo Expos 2013, 2016, 2017, and 2018.

Dr. Sultanian received his BTech and MS in Mechanical Engineering from Indian Institute of Technology, Kanpur and Indian Institute of Technology, Madras, respectively. He received his PhD in Mechanical Engineering from Arizona State University, Tempe and MBA from the Lally School of Management and Technology at Rensselaer Polytechnic Institute.

Physics-Based Modeling of Gas Turbine Secondary Air Systems

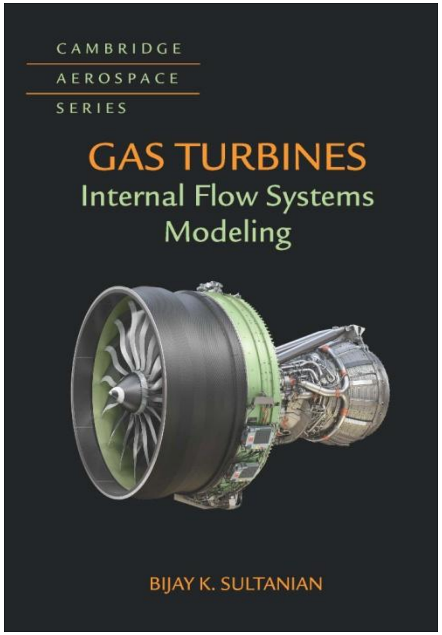
Module 1: An Overview of Secondary Air Systems

Dr. Bijay (BJ) K. Sultanian, PhD, PE, MBA
ASME Life Fellow
Takaniki Communications, LLC
Oviedo, Florida, USA

ASME 2019 TURBO EXPO

Phoenix, Arizona, USA

Sunday, June 16, 2019



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Module 1

An Overview of Secondary Air Systems

Module 1

An Overview of Secondary Air Systems

- Role of Secondary Air Systems (SAS) in gas turbine design engineering
- The concept of physic-based modeling
- Key components of SAS
- Flow network modeling and robust solution techniques
- Role of 3-D CFD in SAS modeling
- Physics-based post-processing of CFD results
- Entropy map generation and application

Some Memorable Quotes (1)

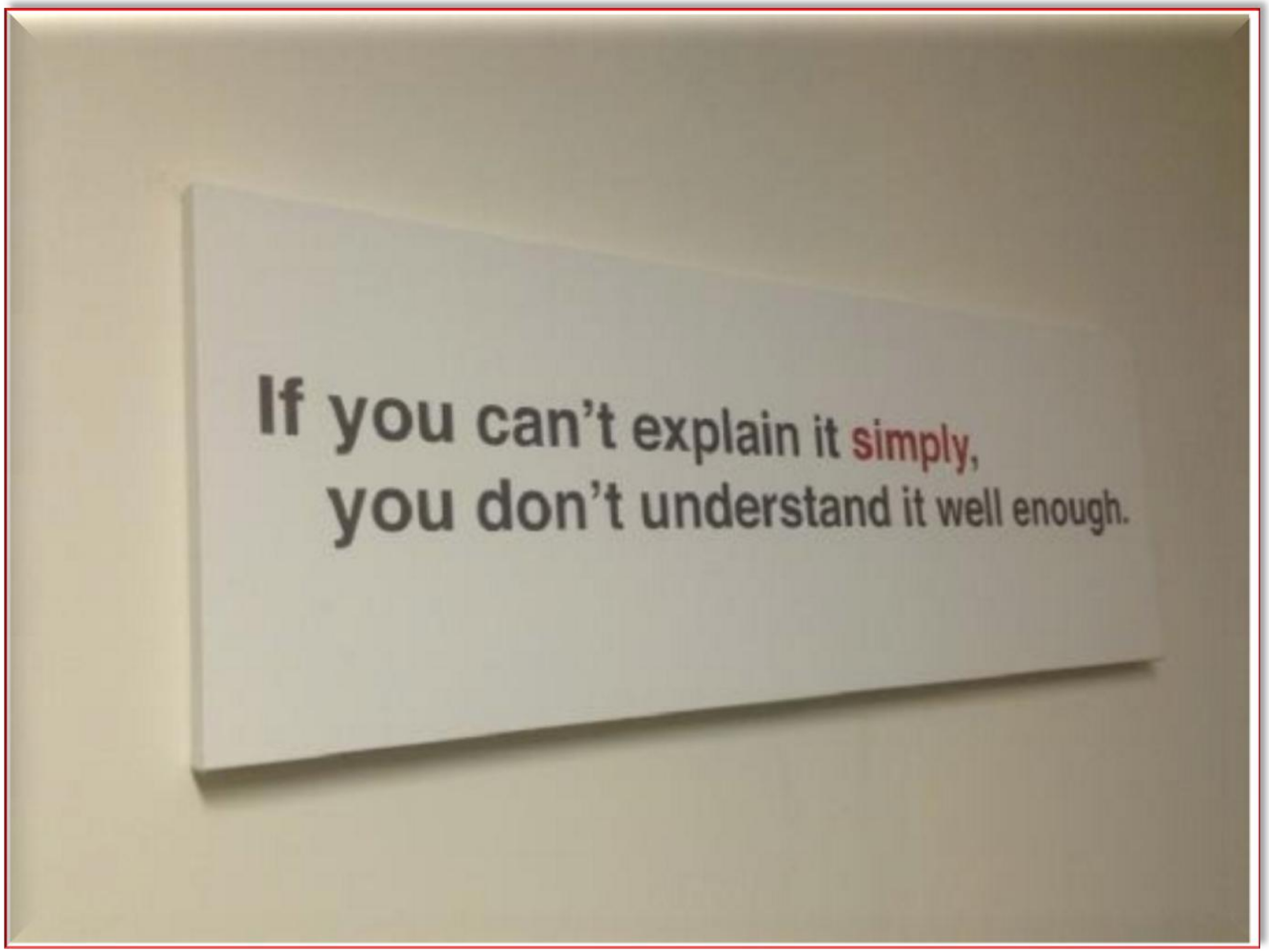
Education is not the learning of facts, but the training of the mind to think.

~ Albert Einstein

Understanding is that penetrating quality of knowledge that grows from theory, practice, conviction, assertion, error, and humiliation.

~ E.B. White

Some Memorable Quotes (2)



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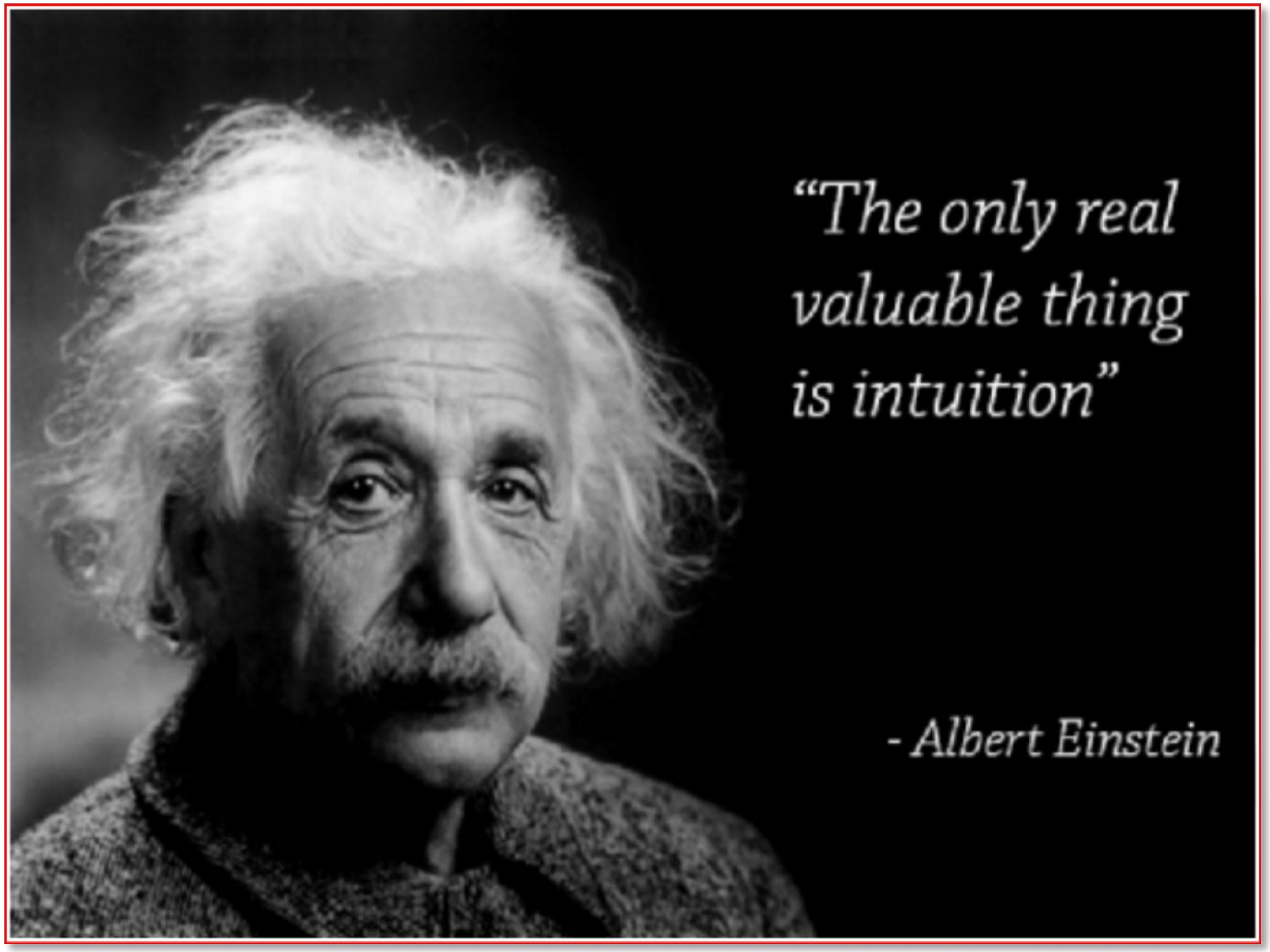
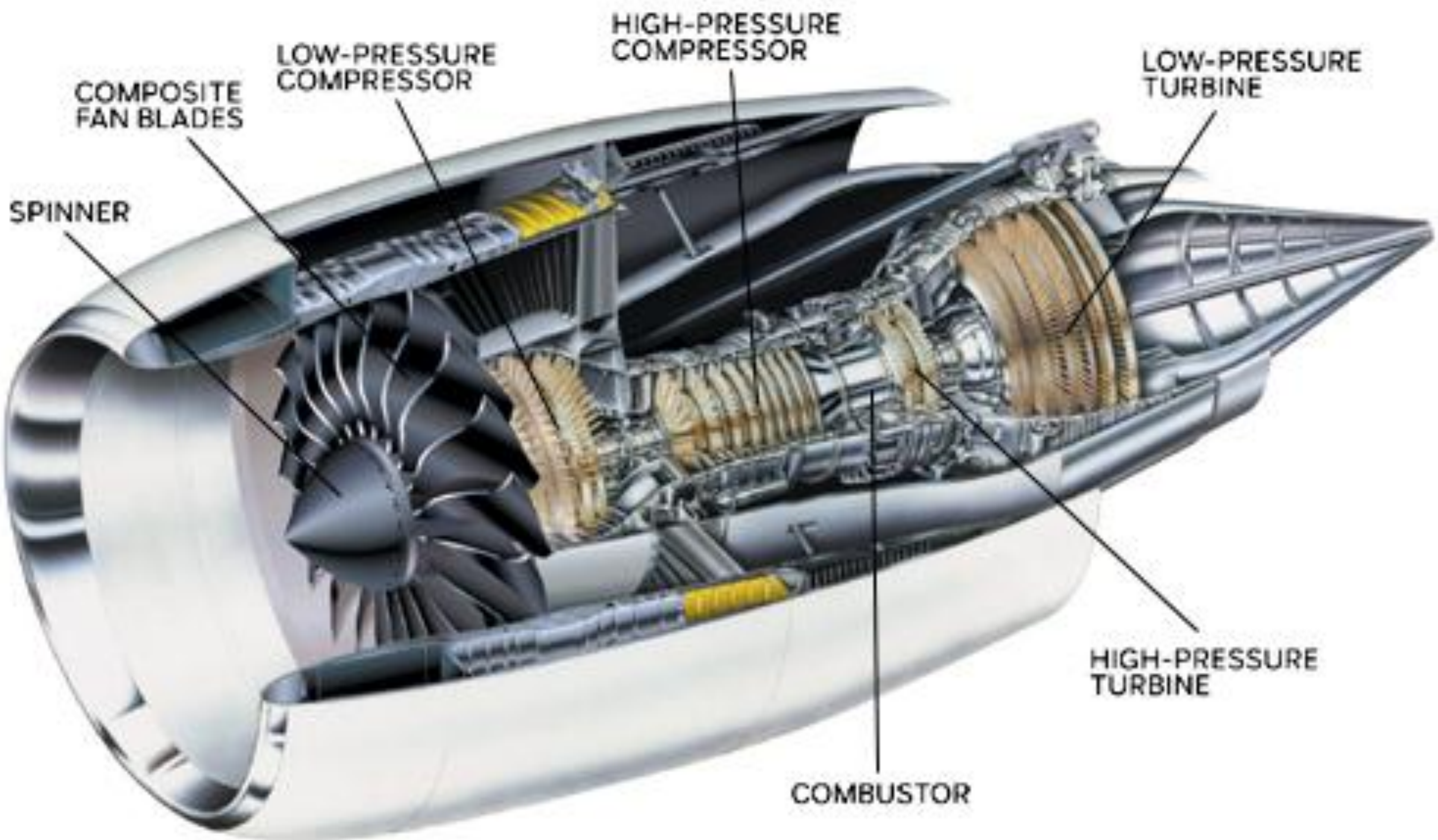
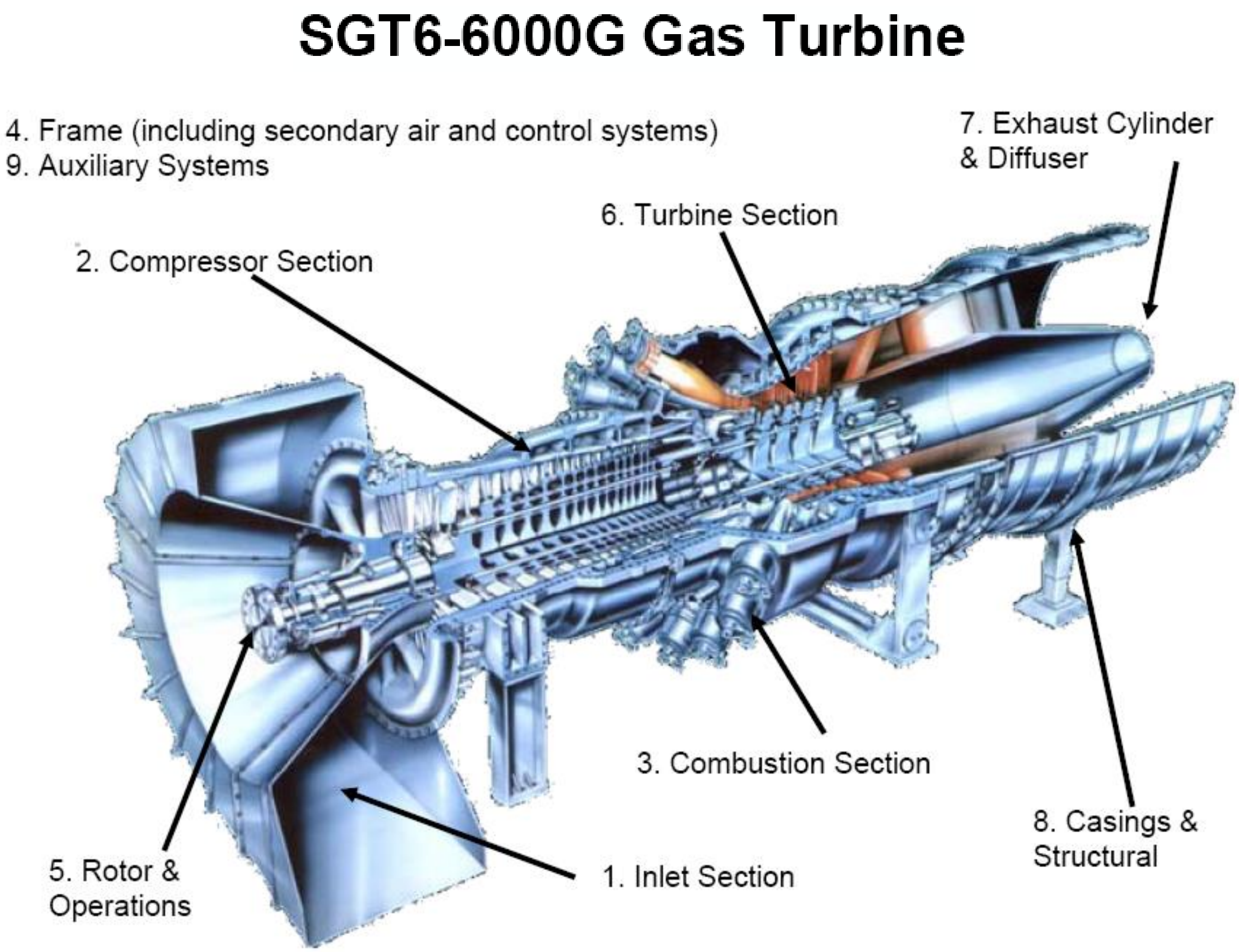
Some Memorable Quotes (3)



An Aircraft Engine

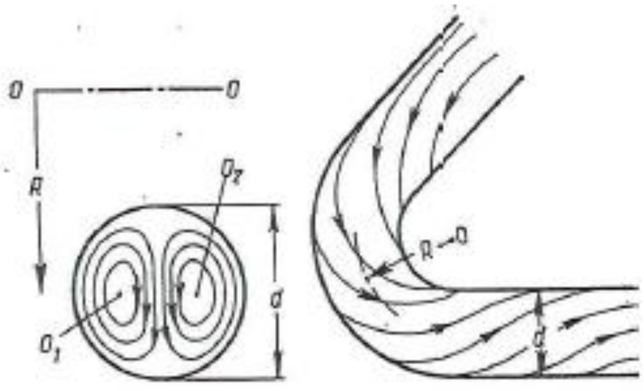


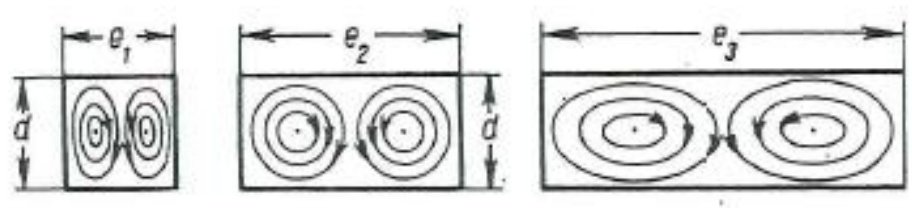
A Gas Turbine Engine for Power Generation



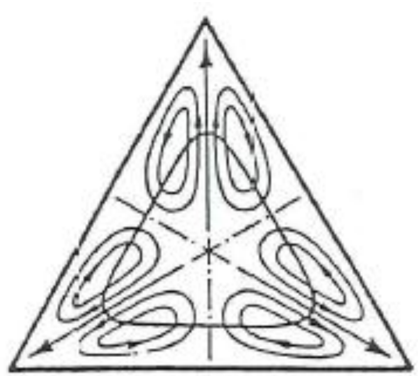
Secondary Flows

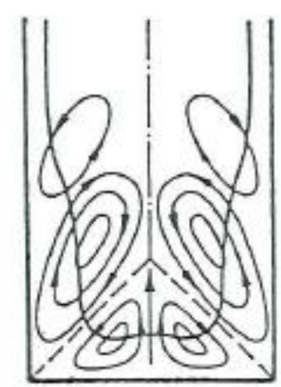
- **Secondary Flow of the First Kind (driven by pressure gradient)**





- **Secondary Flow of the Second Kind (driven by turbulence anisotropy)**





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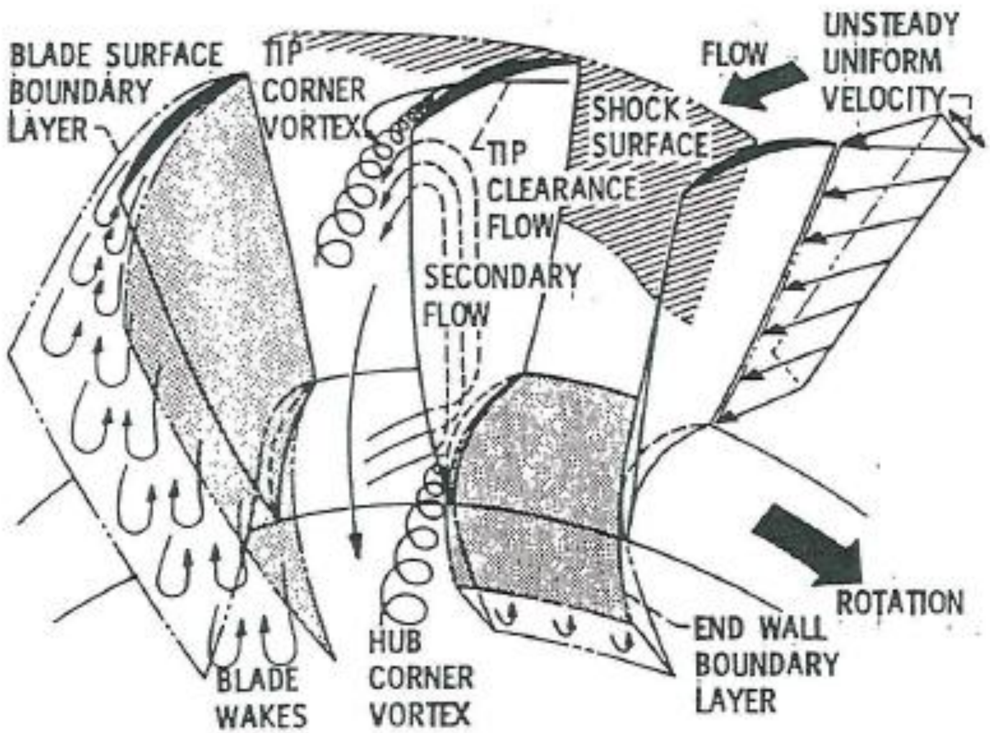
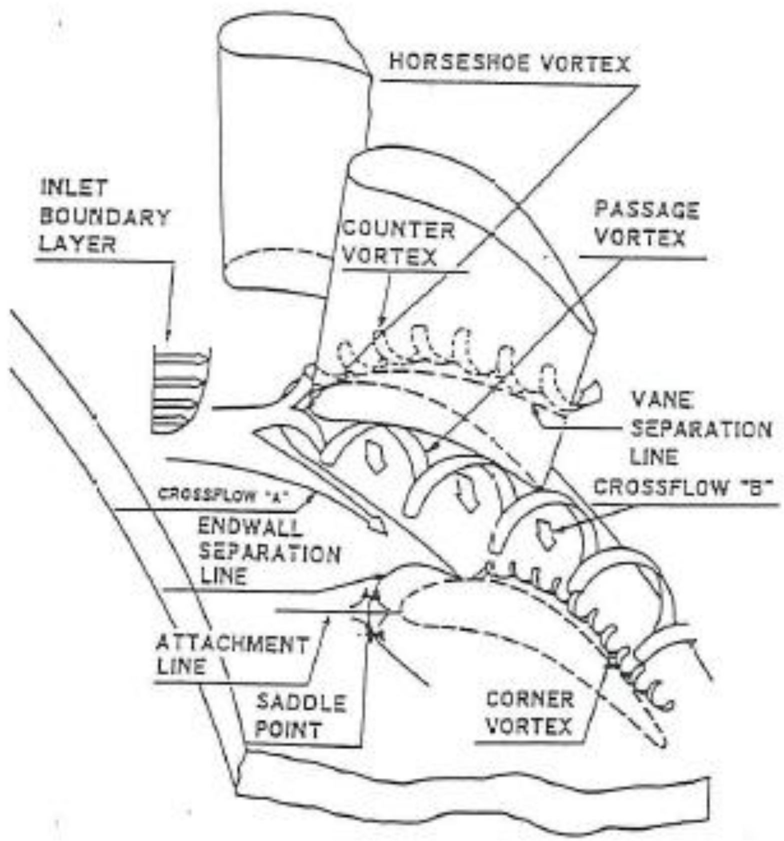
3-D Turbomachinery Aerodynamics (Primary Gas Path)





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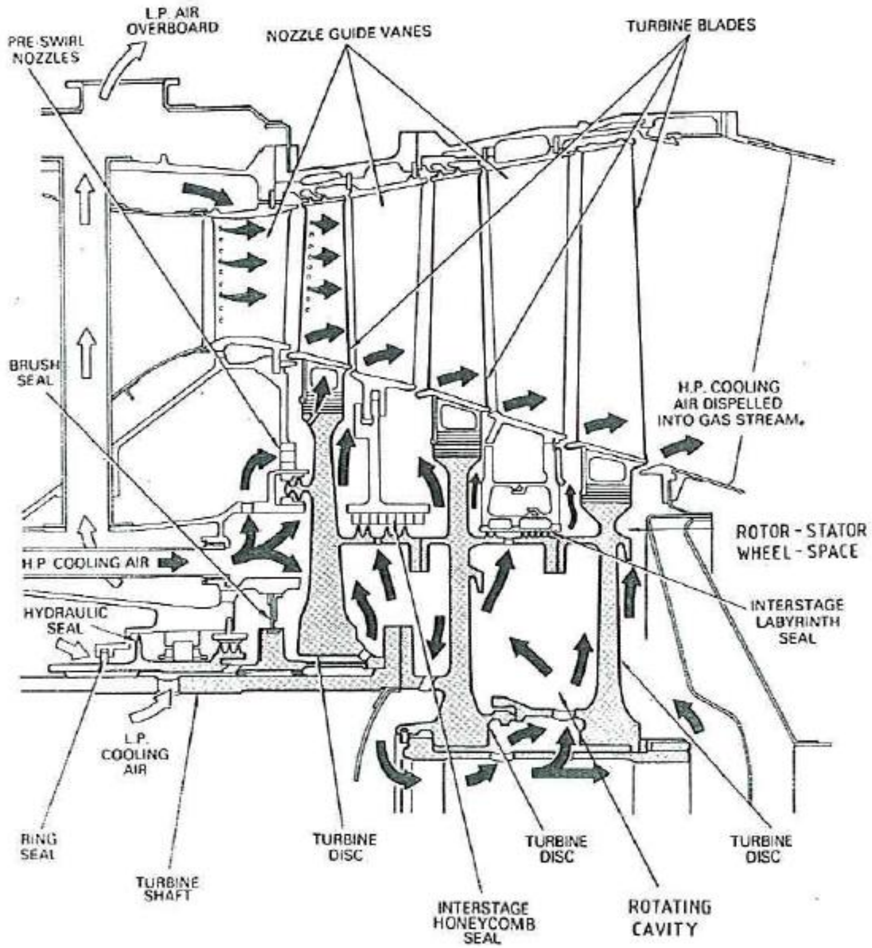
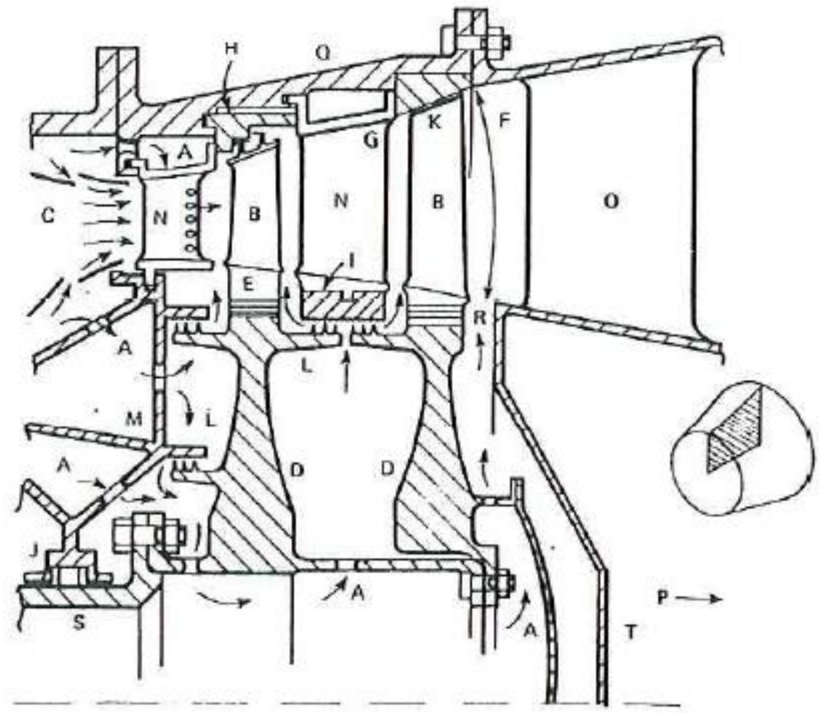
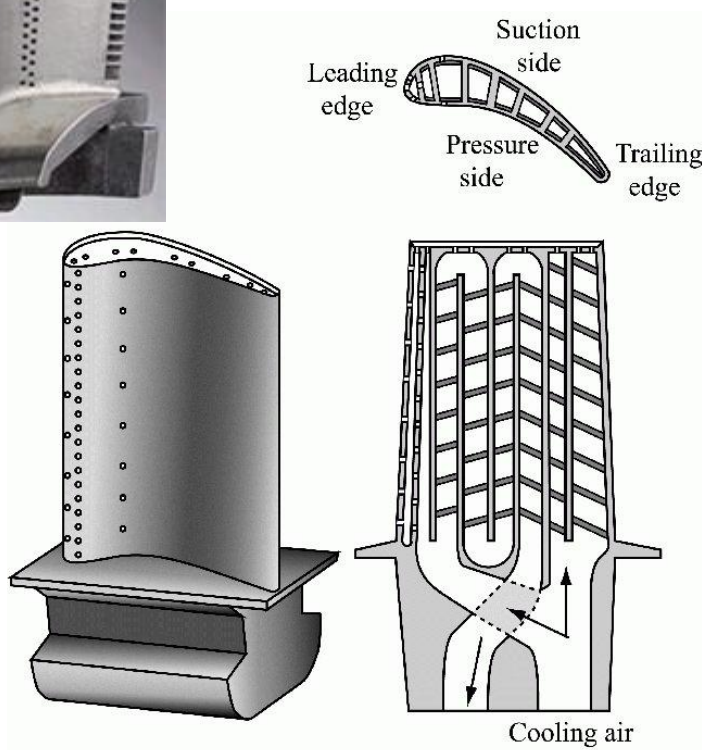
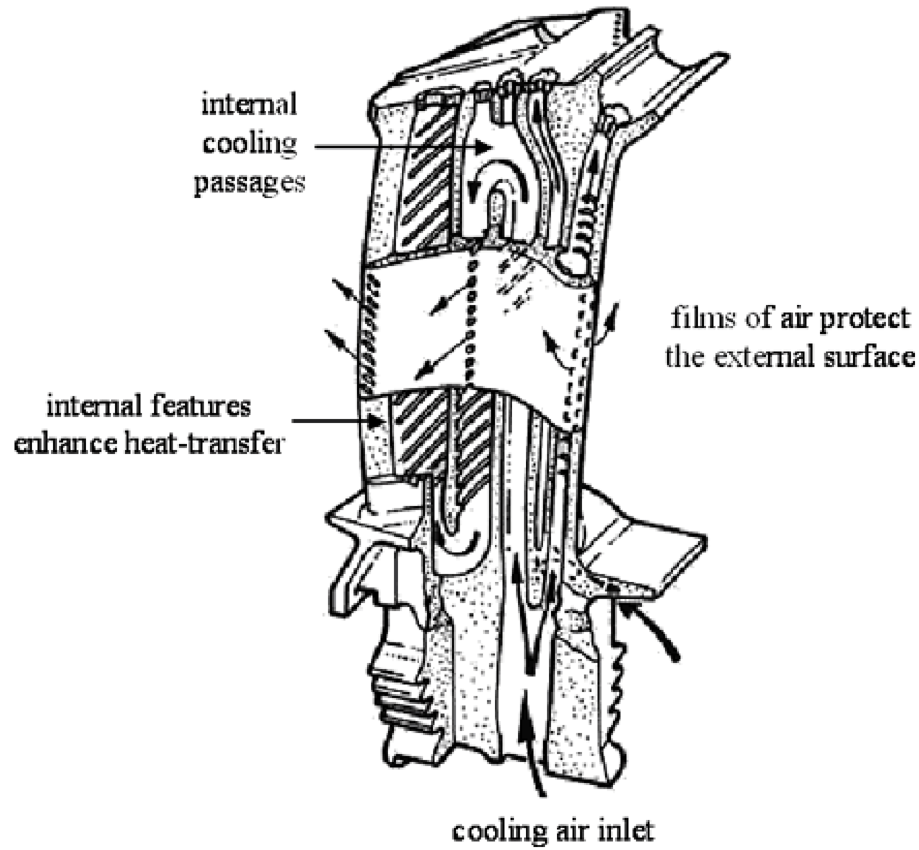
Gas Turbine Cooling and Sealing Systems





Airfoil Internal and Film Cooling (1)





Airfoil Internal and Film Cooling (2)

Shih, T. I-P. and Sultanian, B. K., "Chapter 5: Computations of Internal and Film Cooling," in *Heat Transfer in Gas Turbines*, (Editors: B. Sunden and M. Faghri), WIT Press, Southampton, Boston, 2001, pp. 175-225

Role of Secondary Air Systems (SAS) in Gas Turbine Design Engineering (1)

- Provides cooling flows to various critical components
- Provides sealing flows for bearing chambers and turbine rim seals (to prevent hot gas ingestion)
- Controls bearing axial loads
- Parasitic to the main engine cycle and the energy conversion process associated with the main (primary) flow path
- Involves up to 20% of the engine core flow
- Costs up to 6% of specific fuel consumption

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Role of Secondary Air Systems (SAS) in Gas Turbine Design Engineering (2)

- **Examples:**
  - **An aircraft engine upgrade through SAS redesign**
  - **Competing secondary flow requirements of hot gas ingestion and windage rise in rotor cavities**
  - **Blocker flow in labyrinth seals**
  - **High- and low-pressure inlet bleed heat systems**
  - **Steam-cooled gas turbines**
  - **Steam turbine secondary flow systems**

The Concept of Physics-Based Modeling

- **A physics-based prediction method is formulated using the applicable conservation equations of mass, linear momentum, angular momentum, energy, and entropy over a control volume.**
- **The required auxiliary empirical equations in a physics-based prediction method are in terms of key dimensionless quantities with a broad range of applicability.**
- **The methods entirely based on empirical correlations from model scale experiments tend to be postdictive, and are not considered physics-based.**

Design Prediction Models

- **In engineering, we use many different models for accurate design predictions**

  (e.g., fluid models, flow models, turbulence models, combustion models, etc.)

- **A model that is not <u>understood</u> is a model that will be used in the <u>wrong way</u>!**

Physics-Based Themofluids Modeling

- **Conservations Laws:**
  - **Mass – Continuity Equation**
  - **Linear Momentum**
  - **Angular Momentum**
  - **Energy – First Law of Thermodynamics**
  - **Entropy – Second Law of Thermodynamics**

Physics is the Foundation of Engineering!

Mathematics is the Language of Physics!

3-D DNS & CHT

- **Continuity Equation:** $$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = 0$$
- **Momentum Equations:** $$\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial (\rho u_j u_i)}{\partial x_j} = -\frac{\partial P_s}{\partial x_i} + \frac{\partial \sigma_{ij}}{\partial x_j} + S_i$$
- **Equation of State:** $$\rho = \frac{P_s}{RT_s}$$
- **Energy Equation:** $$\frac{\partial (\rho e)}{\partial t} + \frac{\partial (\rho u_j e)}{\partial x_j} = \frac{\partial \dot{q}_j}{\partial x_j} - \frac{\partial (P u_j)}{\partial x_j} + \frac{\partial (u_i \sigma_{ij})}{\partial x_j} + Q_S$$

$\sigma_{ij} \equiv$ Stress tensor

$\dot{q}_j \equiv$ Diffusive energy flux vector

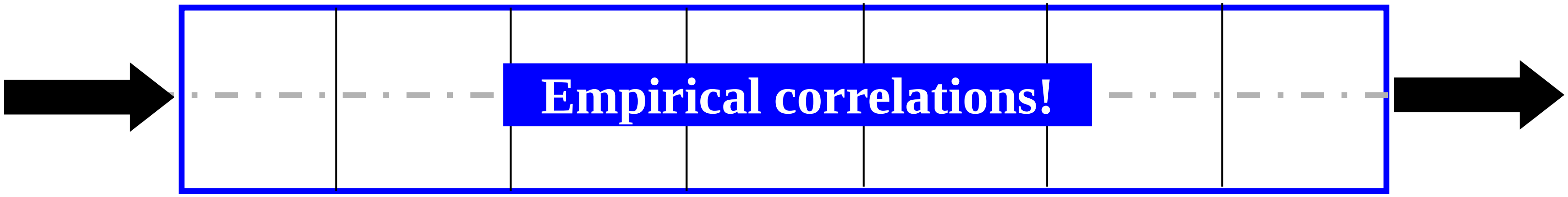
$S_i \equiv$ Momentum source vector

$Q_S \equiv$ Energy source term

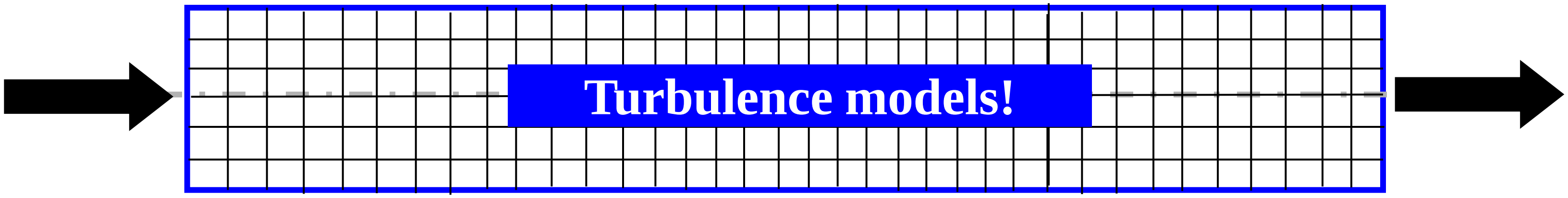
RANS → LES → DNS

Unified Industrial CFD

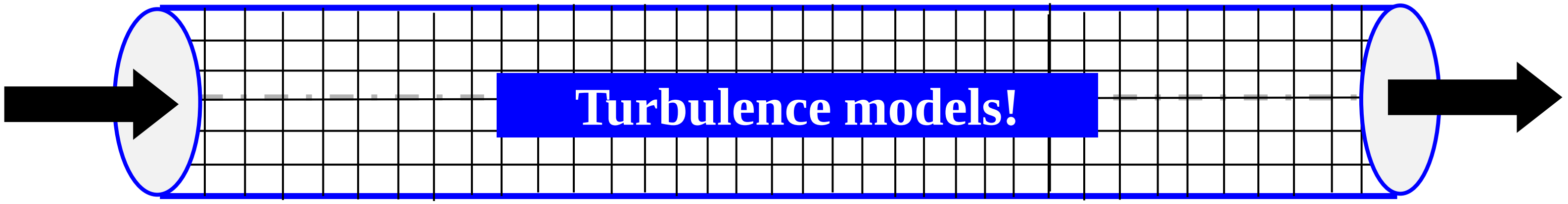
- **1-D CFD:**
  - Integral CV's
  - Each CV has a piece of wall
  - Predicts 1-D variation in properties



- **2-D CFD:**
  - Differential or "small" CV's
  - Most CV's without a wall
  - Predicts 2-D variations in properties



- **3-D CFD:**
  - Differential or "small" CV's
  - Most CV's without a wall
  - Predicts 3-D variations in properties



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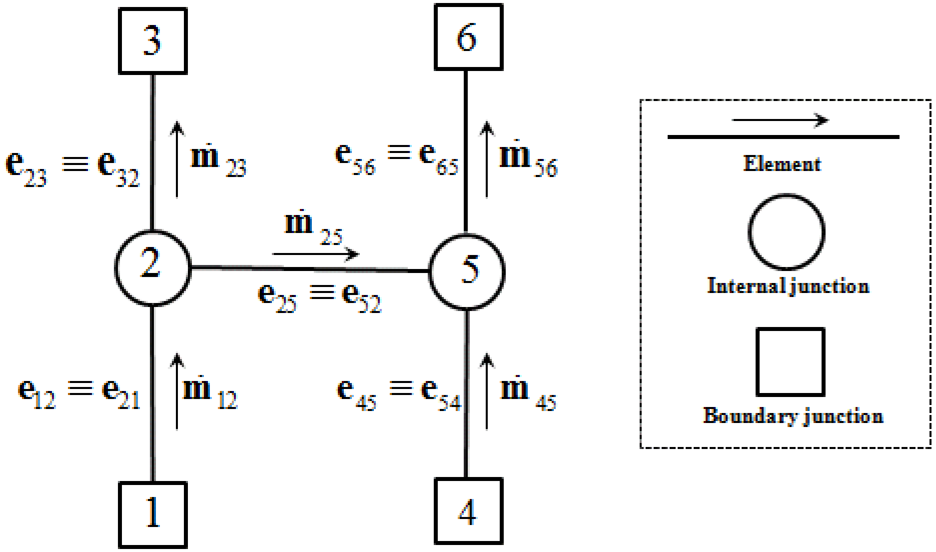
Key Components of SAS

- Orifices (stationary and rotating)
- Channels (stationary and rotating)
- Free and forced vortices
- Seals
- Rotor-rotor and rotor-stator cavities (swirl and windage)
- Nodes (junctions or chambers)
- Interfaces (change of reference frames)

A Flow Network

- A flow network consists of elements (links or branches) and junctions (nodes).
- Each element is characterized by a unique mass flow rate, heat transfer, and work transfer (rotation).
- Each junction is characterized by state variables like pressure, temperature, swirl, etc., and has zero net mass flow rate associated with it (steady state).



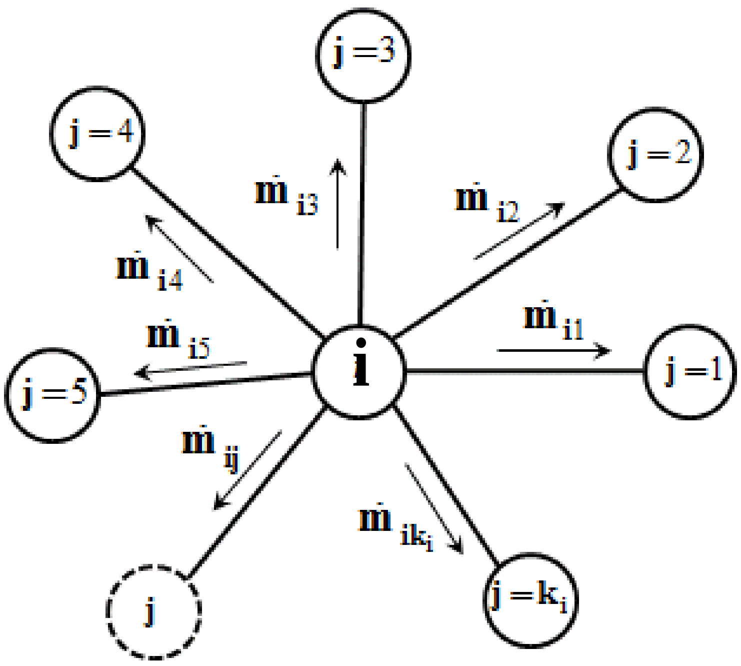
$$\dot{m}_{23} + \dot{m}_{25} = \dot{m}_{12}$$

$$\dot{m}_{12} = -\dot{m}_{21}$$

$$\dot{m}_{23} + \dot{m}_{25} + \dot{m}_{21} = 0$$

Junction Modeling (1)

Continuity Equation



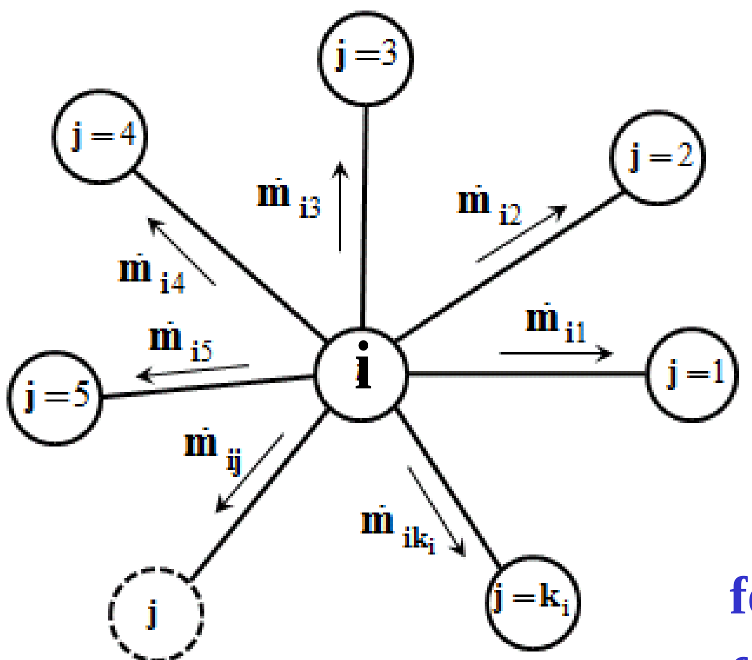
$$\sum_{j=1}^{j=k_i} \dot{m}_{ij} = 0$$

where $j \neq i$

Junction Modeling (2)

Energy Equation

To ensure energy conservation at each internal junction, we can compute the mixed mean total temperature of all incoming flows. All outflows occur at this mixed mean total temperature.



$$\boxed{T_{t_i} = \frac{\sum_{j=1}^{j=k_i} \delta_{ij}\dot{m}_{ij}T_{t_{ij}}}{\sum_{j=1}^{j=k_i} \delta_{ij}\dot{m}_{ij}}}$$

where

$$j \neq i$$

for inflows $(\mathbf{m}_{ij} < 0)$, $\delta_{ij} = 1$

for inflows $(\mathbf{m}_{ij} > 0)$, $\delta_{ij} = 0$

Handling Internal Choking and Normal Shocks

- Compressible flow in a variable-area duct may feature internal choking at a section where **M = 1**.
- If the flow area increases beyond the choked-flow section, the flow becomes supersonic with the possibility of a normal shock if the duct exit conditions are subsonic.
- A good way to simulate the choke-flow section is to make it coincide with a interface between adjacent control volumes.
- For simulating a normal shock, it is better to imbed a thin control volume within which the normal shock occurs.

Note: The flow properties vary continuously across a choked-flow section; however, they vary abruptly across a normal shock.

Determining Element Flow Direction (1)

- To yield accurate solution of a compressible flow network, the flow direction in each element must be determined on physical grounds.
- For an adiabatic flow in an element when one or more effects of area change, friction, and rotation are present, the entropy must always increase in the flow direction.

Thus, for the flow from section 1 to section 2 of an element, we can write

$$\boxed{\frac{s_2 - s_1}{R} = \left(\frac{c_P}{R} \ln \frac{T_{s_2}}{T_{s_1}} - \ln \frac{P_{s_2}}{P_{s_1}} \right) > 0}$$

$$\boxed{\frac{s_2 - s_1}{R} = \left(\frac{c_P}{R} \ln \frac{T_{t_2}}{T_{t_1}} - \ln \frac{P_{t_2}}{P_{t_1}} \right) > 0}$$

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Determining Element Flow Direction (2)

- Heating of the element flow increases its entropy downstream, and cooling decreases it. For applying the entropy-based criterion, it is important to remove the contribution of heat transfer to the total entropy change over the element.

$$hA_w(T_w - \bar{T}_t) = \dot{m}c_p(T_{t_2} - T_{t_1})_{HT}$$

$$\bar{T}_t = T_w - \frac{\dot{m}c_P}{hA_w}(T_{t_2} - T_{t_1})_{HT} = T_w - \frac{(T_{t_2} - T_{t_1})_{HT}}{\eta}$$

where $\bar{T}_t$ is the average total temperature for convective heat transfer over the element.

Determining Element Flow Direction (3)

$$\frac{(\Delta s)_{HT}}{R} = \frac{c_P}{R}\frac{(T_{t_2} - T_{t_1})_{HT}}{\bar{T}_t}$$

Thus, the entropy-based criterion to be used with heat transfer is as follows:

$$\boxed{\frac{s_2 - s_1}{R} - \frac{(\Delta s)_{HT}}{R} = \left(\frac{c_P}{R}\left\{\ln\frac{T_{t_2}}{T_{t_1}} - \frac{(T_{t_2} - T_{t_1})_{HT}}{\bar{T}_t}\right\} - \ln\frac{P_{t_2}}{P_{t_1}}\right) > 0}$$

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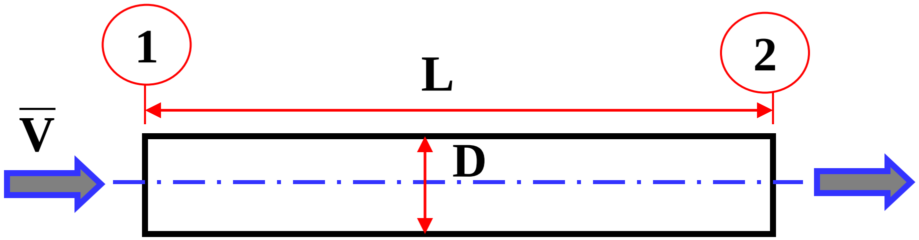
Generating Initial (Starting) Solution

- Hagen-Poiseuille Flow (Fully developed laminar flow in a pipe)



$$\Delta P_s = P_{s_1} - P_{s_2} = f\left(\frac{L}{D}\right)\left(\frac{1}{2}\rho\bar{V}^2\right)$$

$$f = \frac{64}{Re_D} = \frac{64\mu}{\rho\bar{V}D}$$

$$P_{s_1} - P_{s_2} = \left(\frac{64\mu}{\rho\bar{V}D}\right)\left(\frac{L}{D}\right)\left(\frac{1}{2}\rho\bar{V}^2\right) = \left(\frac{32\mu L}{D^2}\right)\bar{V}$$

$$P_{s_1} - P_{s_2} = \left(\frac{32\mu L}{D^2}\right)\bar{V} = \left(\frac{128\mu L}{\rho\pi D^4}\right)\dot{m}$$

$$\dot{m}_{ij} = a_{ij}\left(P_{s_i} - P_{s_j}\right)$$

$$a_{ij} = \frac{\pi}{128}\left(\frac{\rho D^4}{\mu L}\right)$$

- The relation between static pressure drop and mass flow rate is linear!

Robust Iterative Solution Methods (1)

- In a flow network, for an assumed initial junction properties, all the mass flow rates generated in the connected elements will most likely not satisfy the continuity equation at each junction. At an internal junction $\mathbf{i}$ in the flow network, we can thus write

$$\sum_{j=1}^{j=k_i} \dot{m}_{ij} = \sum_{j} \dot{m}_{ij} = \Delta \dot{m}_i$$

where

$$\Delta \dot{m}_i \equiv \text{ mass flow residual at junction } \mathbf{i}$$

In a converged solution, we want to reduce $\Delta \dot{m}_i$ at each junction below a specified tolerance.

Robust Iterative Solution Methods (2)

- For the junction solution, let us assume that the mass flow rate $\dot{m}_{ij}$ through each element e_{ij} depends only on P_{s_i} and P_{s_j}. At each iteration, our goal is to change the junction static pressures so as to annihilate $\Delta\dot{m}_i$. Accordingly, we can write

$$\sum_j d(\dot{m}_{ij}) = d(\Delta\dot{m}_i) = 0 - \Delta\dot{m}_i$$

$$\sum_j \left(\frac{\partial \dot{m}_{ij}}{\partial P_{s_i}} \Delta P_{s_i} + \frac{\partial \dot{m}_{ij}}{\partial P_{s_j}} \Delta P_{s_j} \right) = -\Delta\dot{m}_i$$

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Robust Iterative Solution Methods (3)

From $\dot{m}_{ij} = a_{ij}(P_{s_i} - P_{s_j})$, we obtain $\partial \dot{m}_{ij} / \partial P_{s_i} = a_{ij}$ and $\partial \dot{m}_{ij} / \partial P_{s_j} = -a_{ij}$, giving

$$\boxed{\sum_j (a_{ij}\Delta P_{s_i} - a_{ij}\Delta P_{s_j}) = -\Delta \dot{m}_i}$$

$$\boxed{\begin{bmatrix} \sum_j a_{1j} & \cdot & \cdot & -a_{1n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ -a_{n1} & \cdot & \cdot & \sum_j a_{nj} \end{bmatrix}_{old} \begin{bmatrix} \Delta P_{s_1} \\ \cdot \\ \cdot \\ \Delta P_{s_n} \end{bmatrix}_{new} = \begin{bmatrix} -\Delta \dot{m}_1 \\ \cdot \\ \cdot \\ -\Delta \dot{m}_n \end{bmatrix}_{old}}$$

Robust Iterative Solution Methods (4)

$$\begin{bmatrix} J_{11} & \cdot & \cdot & J_{1n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ J_{n1} & \cdot & \cdot & J_{nn} \end{bmatrix}_{old} \begin{bmatrix} \Delta P_{s_1} \\ \cdot \\ \cdot \\ \Delta P_{s_n} \end{bmatrix}_{new} = \begin{bmatrix} -\Delta \dot{m}_1 \\ \cdot \\ \cdot \\ -\Delta \dot{m}_n \end{bmatrix}_{old}$$

where the coefficient matrix is called the Jacobian matrix:

$$\begin{bmatrix} J_{11} & \cdot & \cdot & J_{1n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ J_{n1} & \cdot & \cdot & J_{nn} \end{bmatrix} = \begin{bmatrix} \sum_j a_{1j} & \cdot & \cdot & -a_{1n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ -a_{n1} & \cdot & \cdot & \sum_j a_{nj} \end{bmatrix}$$

In a typical flow network, Jacobian matrix is generally very sparse with only a handful of non-zero entries.

Robust Iterative Solution Methods (5)

- Newton-Raphson Method

1. Generate an initial distribution of static pressure at each internal junction of the flow network.
2. Compute mass flow rate through each element.
3. Compute Jacobian matrix from element solutions.
4. Compute mass flow rate error at each internal junction.
5. Use a direct solution method to obtain the vector of changes in static pressure at all internal junctions.
6. Obtain the new static pressure at each internal junction:

Robust Iterative Solution Methods (6)

Newton-Raphson Method

$$\left(P_{s_i}\right)_{new} = \left(P_{s_i}\right)_{old} + \alpha_i\left(\Delta P_{s_i}\right)_{new}$$

where α_i is an under-relaxation parameter specified for each junction to help the solution convergence.

7. Repeat steps from 2 to 6 until $\left|\Delta\dot{m}_i\right|_{max} \le \delta_{tol}$ where δ_{tol} is the acceptable maximum error in the mass flow at any internal junction.
8. Solve the energy equation at each internal junction.
9. Repeat steps from 2 to 8 until $\left|(T_{t_i})_{old} - (T_{t_i})_{new}\right|_{max} \le \hat{\delta}_{tol}$ where $\hat{\delta}_{tol}$ is the maximum acceptable difference between the total temperatures at any internal junction in successive iterations.

Robust Iterative Solution Methods (7)

- Modified Newton-Raphson Method

$$\begin{bmatrix} J_{11}+\lambda & \cdot & \cdot & J_{1n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ J_{n1} & \cdot & \cdot & J_{nn}+\lambda \end{bmatrix}_{old} \begin{bmatrix} \Delta P_{s_1} \\ \cdot \\ \cdot \\ \Delta P_{s_n} \end{bmatrix}_{new} = \begin{bmatrix} -\Delta\dot{m}_1 \\ \cdot \\ \cdot \\ -\Delta\dot{m}_n \end{bmatrix}_{old}$$

where the damping parameter λ, which is auto-adjusted during the iterative solution process, ensures that the Jacobian matrix remains diagonally dominant, preventing it from becoming singular.

Robust Iterative Solution Methods (8)

Modified Newton-Raphson Method

1. Generate an initial distribution of static pressure at each internal junction of the flow network.
2. Set $\lambda_{old} = 0.5$ and $\lambda_{new} = \lambda_{old}$.
3. Compute mass flow rate through each element.
4. Compute residual error of the continuity equation at each internal junction and the corresponding error norm

$$\boxed{E_{old} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\Delta \dot{m}_i)^2}}$$

Robust Iterative Solution Methods (9)

Modified Newton-Raphson Method

5. Compute Jacobian matrix from solution for each element in the flow network.
6. Modify the Jacobian matrix by adding to all its diagonal components.
7. Use a direct solution method to obtain the vector of changes in static pressure at all internal junctions.
8. Obtain the new static pressure at each internal junction:

$$\boxed{\left(P_{s_i}\right)_{new} = \left(P_{s_i}\right)_{old} + \left(\Delta P_{s_i}\right)_{new}}$$

9. Compute mass flow rate through each element.

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Robust Iterative Solution Methods (10)

Modified Newton-Raphson Method

10. Compute error of the continuity equation at each internal junction and the error norm

$$E_{new} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\Delta\dot{m}_i)^2}$$

11. If $E_{new} > E_{old}$, set $\lambda_{new} = 2\lambda_{old}$ and repeat steps from 6 to 11.

12. Set $\lambda_{new} = \lambda_{old}/2$

13. Repeat steps from 5 to 12 until $E_{new} \le \tilde{\delta}_{tol}$, where $\tilde{\delta}_{tol}$ is the maximum acceptable value of E_{new}.

Robust Iterative Solution Methods (11)

- **Modified Newton-Raphson Method**

14. Solve the energy equation at each internal junction.

15. Repeat steps from 5 to 14 until $\left|(T_{t_i})_{old} - (T_{t_i})_{new}\right|_{max} \leq \hat{\delta}_{tol}$, where $\hat{\delta}_{tol}$ is the maximum acceptable difference between the total temperatures at any internal junction in successive iterations.

Note:

Modified Newton-Raphson method provides a robust alternative to the standard Newton-Raphson method, which is sensitive to initial solution, often requiring adjustments of under-relaxation parameters to obtain a converged solution of the flow network.

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Role of 3-D CFD in SAS Modeling (1)

❑ Definition of CFD

CFD is the numerical prediction of the distributions of velocity, pressure, temperature, concentration, and other relevant variables throughout the calculation domain

CFD Offers A Physics-Based 3-D Prediction Method.

Role of 3-D CFD in SAS Modeling (2)

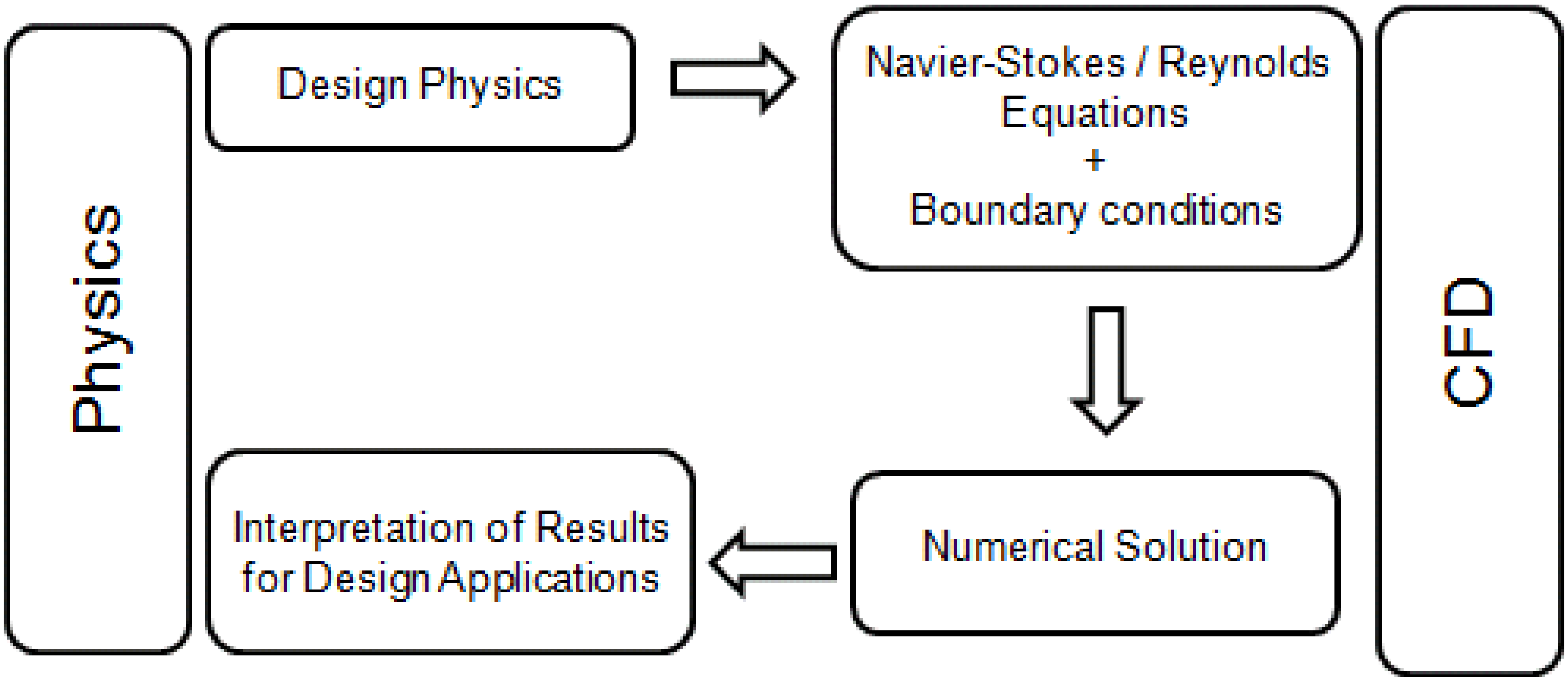
- The Conservation Equations in a Common Form

$$\underbrace{\frac{\partial}{\partial t}(\varrho\Phi)}_{\textit{Transient}} + \underbrace{\frac{\partial}{\partial x_j}(\varrho U_j\Phi)}_{\textit{Convection}} = \underbrace{\frac{\partial}{\partial x_j}\left(\Gamma_\Phi\frac{\partial\Phi}{\partial x_j}\right)}_{\textit{Diffusion}} + \underbrace{S_\Phi}_{\textit{Source}}$$

$$\Phi = 1,\ U_i,\ h,\ and\ m_l$$

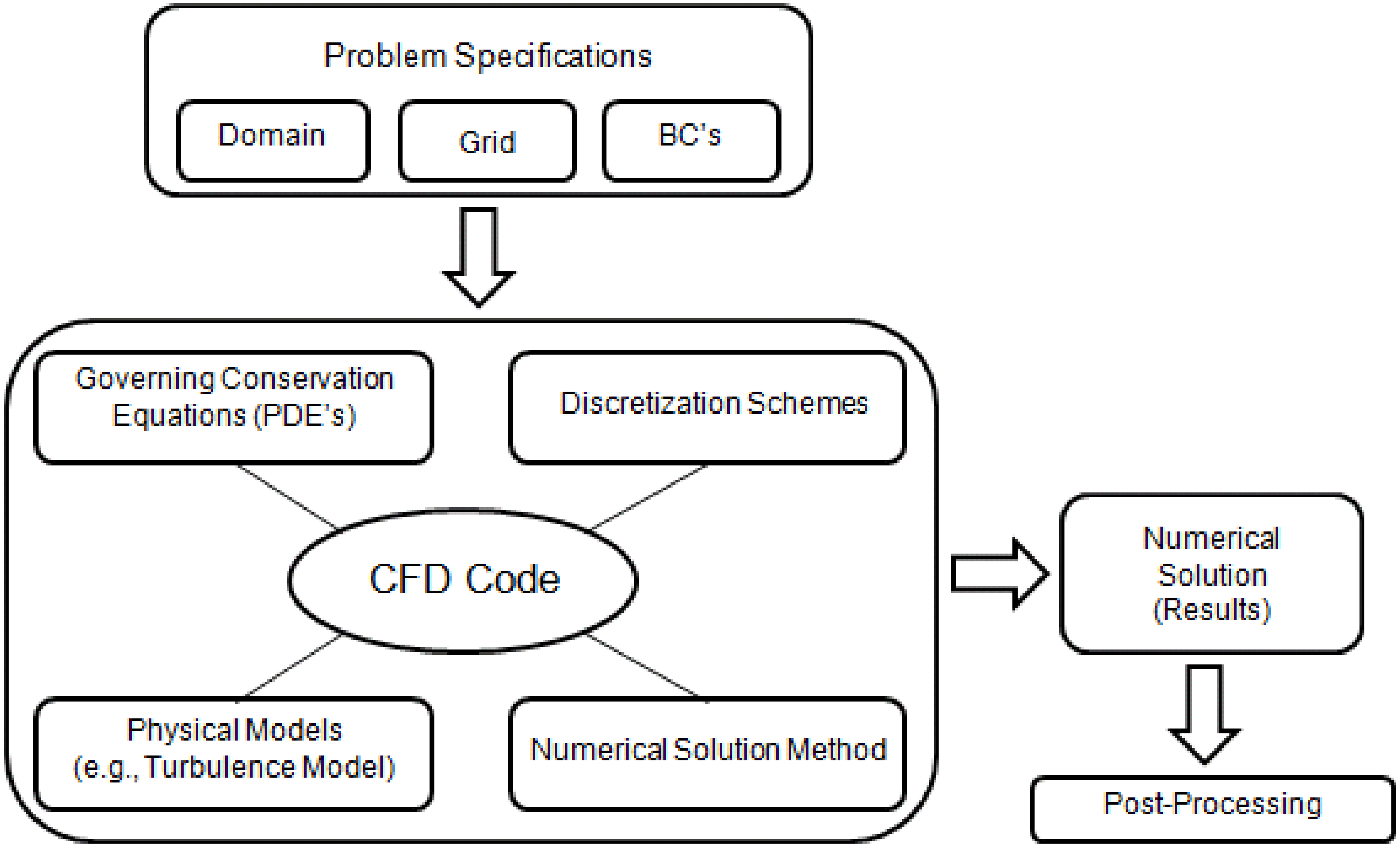
Role of 3-D CFD in SAS Modeling (3)

- CFD Modeling Methodology



Role of 3-D CFD in SAS Modeling (4)

- CFD Using a Commercial Code

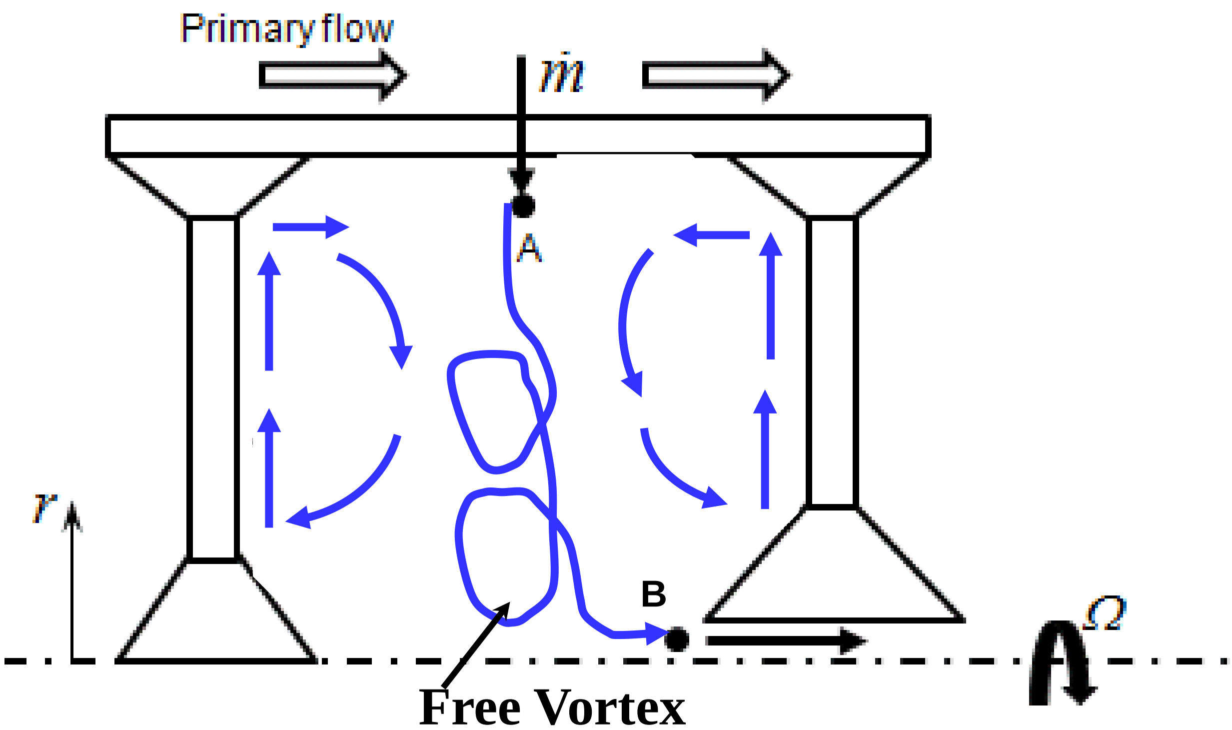


Role of 3-D CFD in SAS Modeling (5)

- 3-D CFD yields detailed micro-analysis results
- SAS Modeling yields macro-analysis results
- 3-D CFD analysis is an excellent means of quick and cheap flow visualization
- 3-D CFD results can guide simplified SAS modeling of a new component / flow field

Role of 3-D CFD in SAS Modeling (6)

An Example



- Intuitive flow field schematic in a compressor rotor cavity with radial inflow

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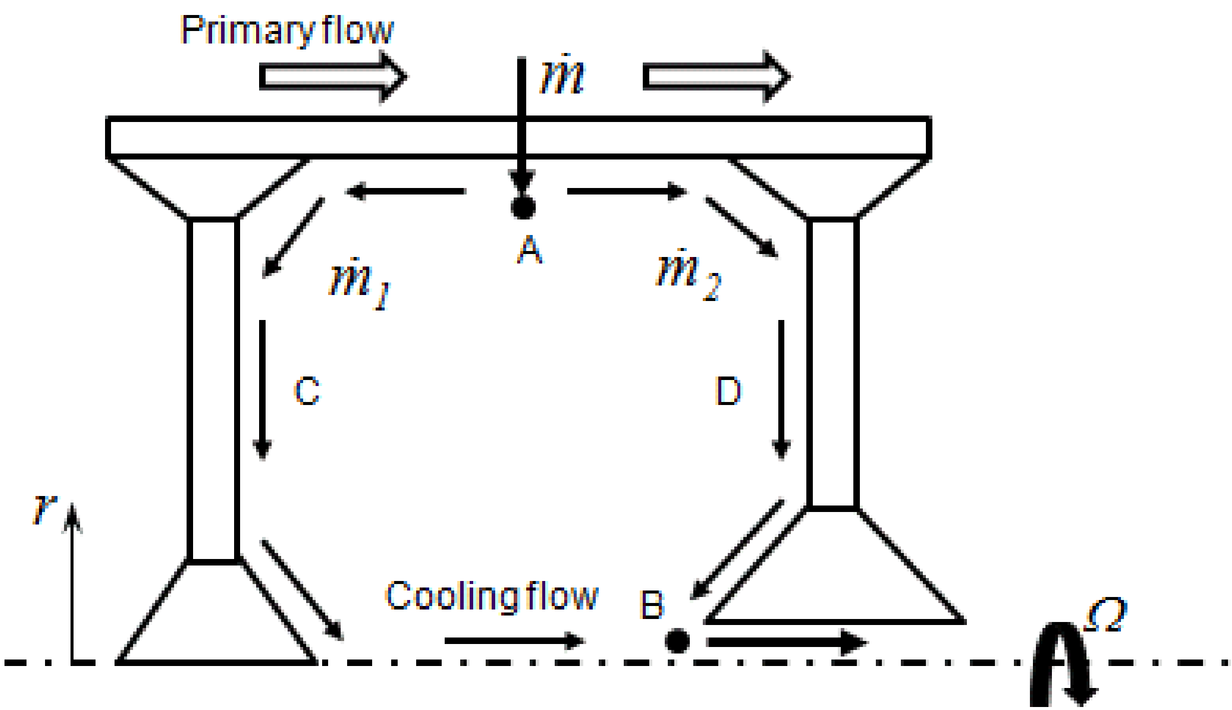
Role of 3-D CFD in SAS Modeling (7)

An Example



- Counter-intuitive flow field schematic in a compressor rotor cavity with radial inflow

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Physics-Based Post-Processing of CFD Results (1)

- CFD computes for each cell face or node the following primitive variables of the flow field:

$$V_x, V_y, V_z, P_s, h_s \text{ (or } T_s\text{)}, \rho$$

- Two key requirements for a section-averaged quantity:

1. **Averaging must preserve the total flow of mass, momentum, angular momentum, energy, and entropy through the section and total surface force or torque at the section given by the CFD solution.**
2. **Assuming the section to be a control surface, the computed section-averaged quantity must be usable in an integral control volume analysis.**

Physics-Based Post-Processing of CFD Results (2)

- **Without any loss of generality, the section is assumed to be normal to x-axis (main flow direction).**

- **Step 1: Compute mass flow rate through the section**

$$\dot{m} = \iint_A \rho V_x dA$$

Note: V_x can be either positive or negative.

- **Step 2: Compute flow of static enthalpy through the section**

$$\dot{h}_s = \iint_A h_s \rho V_x dA$$

Note: V_x can be either positive or negative.

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Physics-Based Post-Processing of CFD Results (3)

- **Step 3:** Compute section-average static enthalpy

$$\boxed{\tilde{h}_s = \frac{\dot{h}_s}{\dot{m}}}$$

- **Step 4:** Using property tables or functions, compute section-average static temperature and specific heat at constant pressure

$$\boxed{\tilde{h}_s \rightarrow \tilde{T}_s \text{ and } \tilde{c}_P}$$

- **Step 5:** Compute flow of kinetic energy through the section

$$\dot{E}_{KE} = \iint_A \left(\frac{V^2}{2}\right) \rho V_x dA$$

Note: V_x can be either positive or negative.

Physics-Based Post-Processing of CFD Results (4)

- **Step 6:** **Compute section-average specific kinetic energy**

$$\boxed{\frac{\tilde{V}^2}{2} = \frac{\dot{E}_{KE}}{\dot{m}}}$$

- **Step 7:** **Compute section-average total temperature**

$$\boxed{\tilde{T}_t = \tilde{T}_s + \frac{\tilde{V}^2}{2\tilde{c}_P}}$$

- **Step 8:** **Compute section-average ratio of specific heats**

$$\boxed{\tilde{\kappa} = \frac{\tilde{c}_P}{\tilde{c}_P - R}}$$

$R \equiv$ **Gas constant**

Physics-Based Post-Processing of CFD Results (5)

- **Step 9: Compute section-average static pressure**

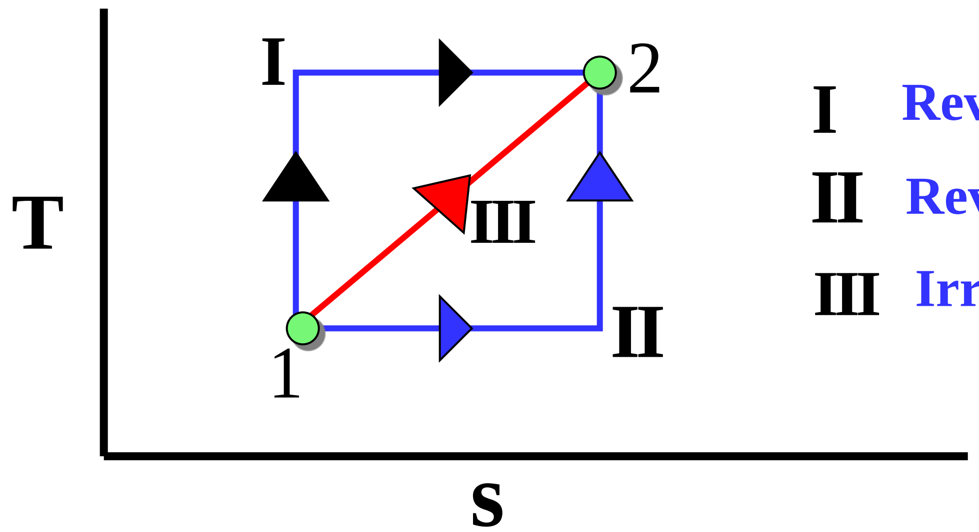
$$\boxed{\tilde{P}_s = \frac{\iint_A P_s\, dA}{\iint_A dA}}$$

Note: $\tilde{P}_s$ **is an area-average quantity.**

- **Step 10: Compute section-average total pressure**

$$\boxed{\tilde{P}_t = \tilde{P}_s \left(\frac{\tilde{T}_t}{\tilde{T}_s} \right)^{\frac{\tilde{\kappa}}{\tilde{\kappa}-1}}}$$

What Is Entropy?



I Reversible process

II Reversible process

III Irreversible process

$$\left[ds\right]_1^2 = \left[\int_1^2 \frac{dq_{rev}}{T_s}\right]_{\text{Path I}} = \left[\int_1^2 \frac{dq_{rev}}{T_s}\right]_{\text{Path II}} > \left[\int_1^2 \frac{dq_{irrev}}{T_s}\right]_{\text{Path III}}$$

Computing Change in Entropy (1)

- **The First Law of Thermodynamics**
  - **In terms of specific enthalpy (h_s)**

$$\boxed{dh_s = \delta q - \delta u_f + \frac{dP_s}{\rho}}$$

For a reversible process $du_f = 0$, giving

$$\boxed{dh_s = \delta q_{rev} + \frac{dP_s}{\rho}}$$

Computing Change in Entropy (2)

- The Second Law of Thermodynamics

$$\boxed{ds = \frac{\delta q_{rev}}{T_s}}$$

Combining the first law (in terms of enthalpy) and the second law of thermodynamics yields

$$dh_s = T_s ds + \frac{dP_s}{\rho} \quad \text{or} \quad ds = \frac{dh_s}{T_s} - \frac{dP_s}{\rho T_s}$$

For a calorically perfect gas, we obtain

$$\boxed{ds = c_P \frac{dT_s}{T_s} - R \frac{dP_s}{P_s}}$$

Computing Change in Entropy (3)

Now

$$\int_1^2 ds = c_P \int_1^2 \frac{dT_s}{T_s} - R\int_1^2 \frac{dP_s}{P_s}$$

or

$$\boxed{s_2 - s_1 = c_P \ln\left(\frac{T_{s_2}}{T_{s_1}}\right) - R\ln\left(\frac{P_{s_2}}{P_{s_1}}\right)}$$

or

$$\boxed{\frac{P_{s_2}}{P_{s_1}} = \left(\frac{T_{s_2}}{T_{s_1}}\right)^{\frac{c_P}{R}} e^{-\frac{(s_2-s_1)}{R}} = \left(\frac{T_{s_2}}{T_{s_1}}\right)^{\frac{\kappa}{\kappa-1}} e^{-\frac{(s_2-s_1)}{R}}}$$

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Computing Change in Entropy (4)

- For an isentropic process

$$\boxed{s_2 - s_1 = 0} \quad \Rightarrow \quad \boxed{c_P \ln\left(\frac{T_{s_2}}{T_{s_1}}\right) = R \ln\left(\frac{P_{s_2}}{P_{s_1}}\right)}$$

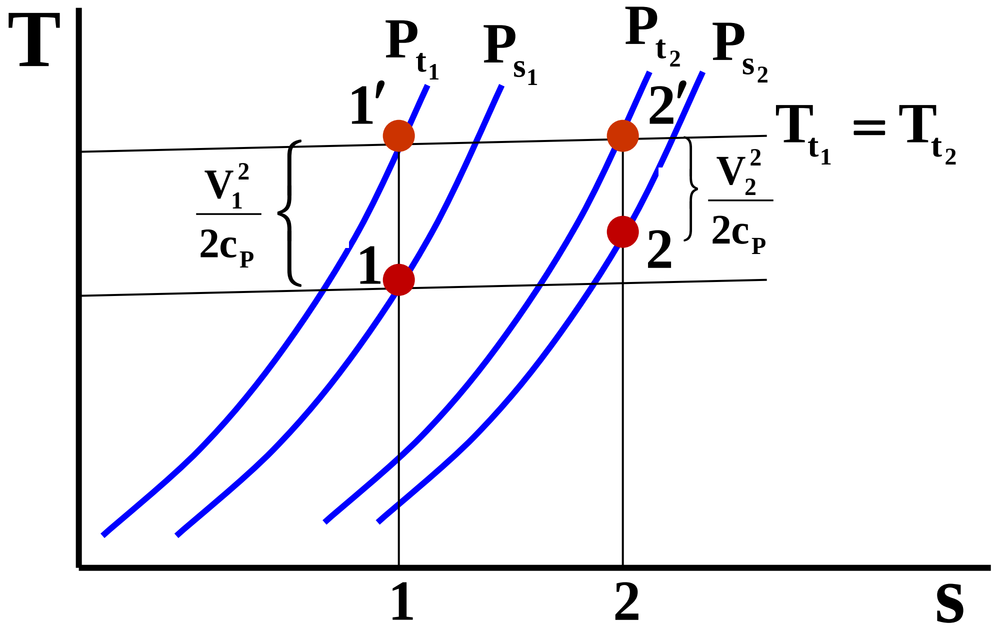
or

$$\boxed{\frac{P_{s_2}}{P_{s_1}} = \left(\frac{T_{s_2}}{T_{s_1}}\right)^{\frac{\kappa}{\kappa-1}}}$$

Using the equation of state, we obtain

$$\boxed{\frac{P}{\rho^{\kappa}} = \text{Constant}} \quad \text{and} \quad \boxed{\frac{\rho_{s_2}}{\rho_{s_1}} = \left(\frac{P_{s_2}}{P_{s_1}}\right)^{\frac{1}{\kappa}} = \left(\frac{T_{s_2}}{T_{s_1}}\right)^{\frac{1}{\kappa-1}}}$$

Entropy Change in Terms of Total Properties (1)

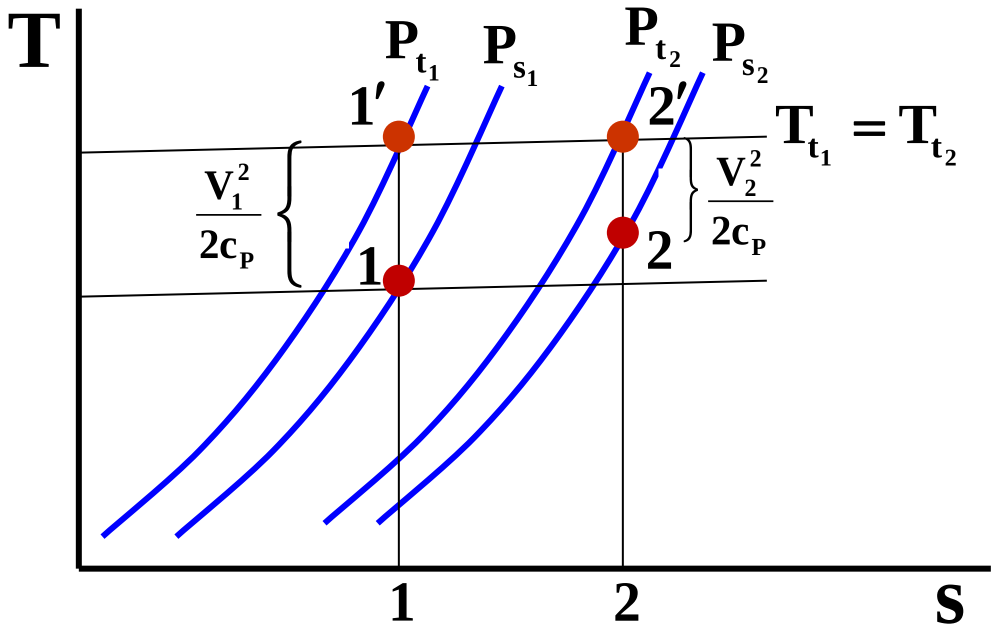


$$s_2 - s_1 = c_P \ln\left(\frac{T_{s_2}}{T_{s_1}}\right) - R\ln\left(\frac{P_{s_2}}{P_{s_1}}\right)$$

$$s_{2'} - s_{1'} = c_P \ln\left(\frac{T_{t_2}}{T_{t_1}}\right) - R\ln\left(\frac{P_{t_2}}{P_{t_1}}\right)$$

$$\boxed{P_{s_2} < P_{s_1}} \qquad \boxed{T_{s_2} > T_{s_1}}$$

Entropy Change in Terms of Total Properties (2)



Note:

$$s_{2'} - s_{1'} = s_2 - s_1$$

For $T_{t1} = T_{t2}$ we obtain

$$\boxed{s_2 - s_1 = -R \ln\left(\frac{P_{t_2}}{P_{t_1}}\right)}$$

Thus, for an adiabatic process $s_2 > s_1$ **and** $P_{t_2} < P_{t_1}$

Dimensionless Entropy

❖ Dimensionless Local Specific Entropy (s^*)

$$s_2 - s_1 = c_P \ln\left(\frac{T_{s\,2}}{T_{s\,1}}\right) - R \ln\left(\frac{P_{s\,2}}{P_{s\,1}}\right)$$

$$s^* = \frac{s - s_{ref}}{R} = \ln\left(\frac{\left(\frac{T_s}{T_{s\,ref}}\right)^{\frac{c_p}{R}}}{\left(\frac{P_s}{P_{s\,ref}}\right)}\right) = \ln\left(\frac{\left(\frac{T_s}{T_{s\,ref}}\right)^{\frac{\kappa}{\kappa-1}}}{\left(\frac{P_s}{P_{s\,ref}}\right)}\right)$$

Entropy Map Generation and Interpretation

- **Step 1:** Obtain 3-D CFD results in the domain of design interest.

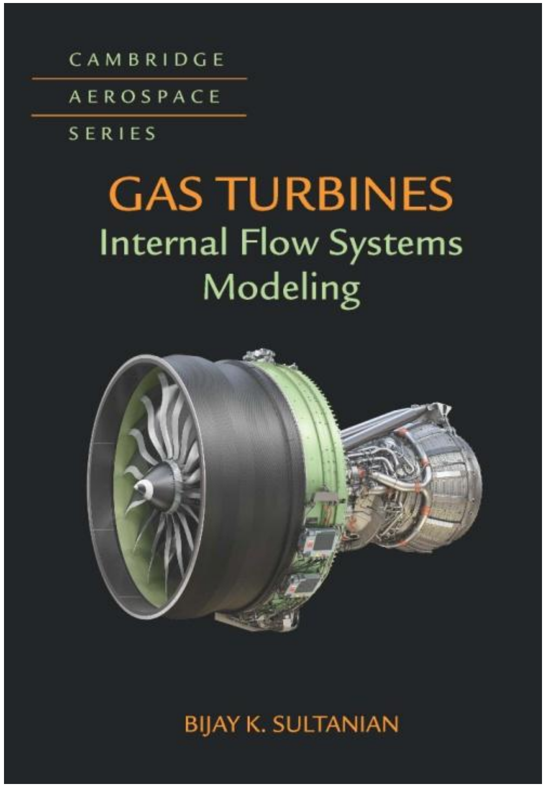
- **Step 2:** Compute $\bar{s}^*_{inlet}$, which is the average value of s^* at the domain inlet.

- **Step 3:** Post-process 3-D CFD results to compute $(s^* - \bar{s}^*_{inlet})$ whose contour plot becomes the domain entropy map.

- **Step 4:** The regions of high entropy production are the regions to be reduced or eliminated in the next design iteration.

QUESTIONS?

THANK YOU!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Internal Flow Systems
Modeling
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Physics-Based Modeling of Gas Turbine Secondary Air Systems

Module 2: Special Concepts of Secondary Air Systems – Part I

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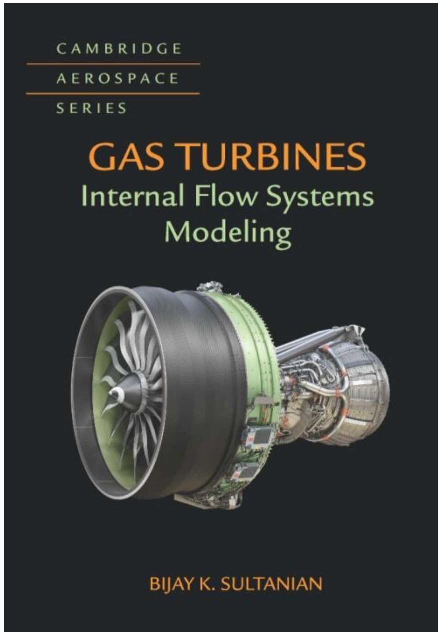

ASME 2019 TURBO EXPO
Phoenix, Arizona, USA
Sunday, June 16, 2019

Module 2

Special Concepts of Secondary Air Systems – Part I

Module 2

- Special Concepts of Secondary Air Systems – Part I
  - Free vortex
  - Forced vortex
  - Rankine vortex
  - Windage
  - Compressible mass flow functions
  - Loss coefficient and discharge coefficient for an incompressible flow
  - Loss coefficient and discharge coefficient for a compressible flow

Vortex

Among the vortices is one which is slower at the center than at the sides, another faster at the center than at the sides.

— Leonardo da Vinci

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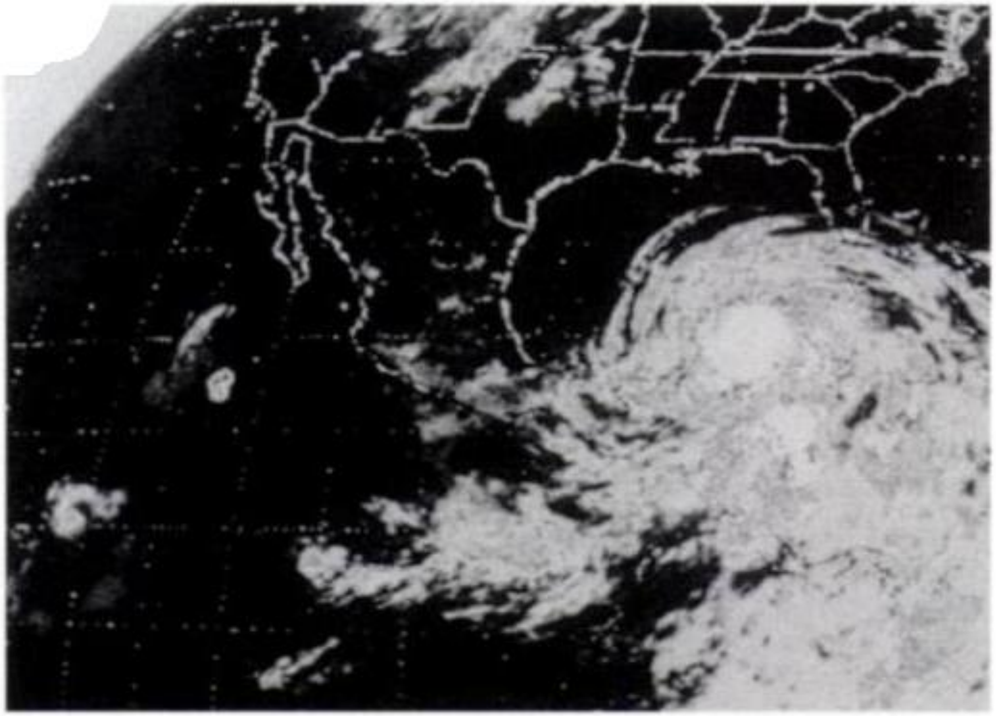
Free Vortex in Nature

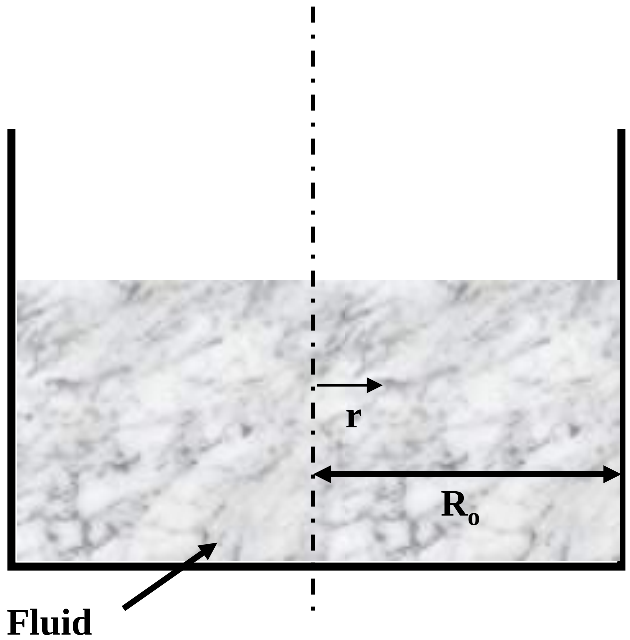
- A Tornado Funnel

- An Infrared Satellite Photo of Hurricane Gilbert
  (14 September 1988; NASA Plate)

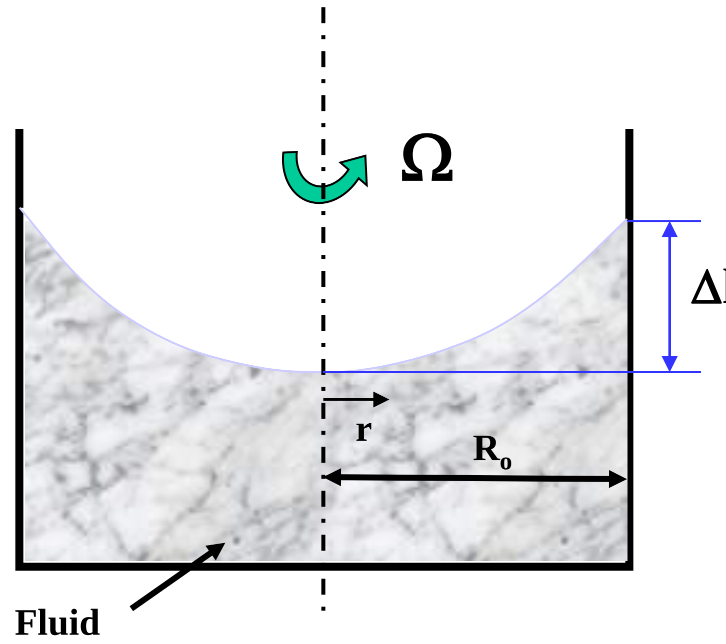


Forced Vortex

- Initial State without Rotation



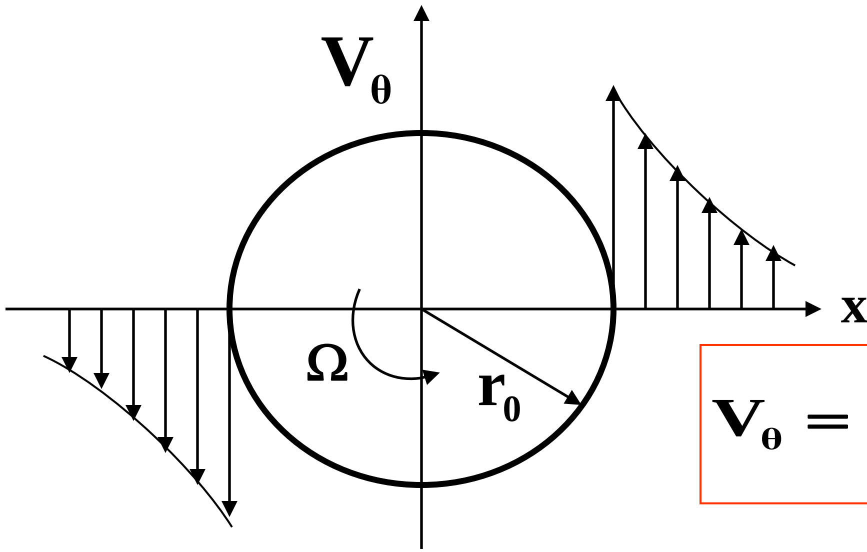
- Solid Body Rotation



$$\Delta h = \frac{R_o^2 \Omega^2}{2g}$$

Vortex (1)

Free Vortex (Potential Vortex)



$$V_\theta = \frac{C}{r}$$

$$V_\theta = \frac{C}{r_0} = r_0\Omega$$

$$C = r_0^2\Omega$$

- Cartesian coordinates:

$$\vec{V} = -\frac{Cy}{x^2+y^2}\hat{i} + \frac{Cx}{x^2+y^2}\hat{j}$$

- Cylindrical coordinates:

$$\vec{V} = \frac{C}{r}\hat{e}_\theta$$

Vortex (2)

- Free Vortex (Potential Vortex)
  - Cartesian coordinates

$$\vec{V} = -\frac{Cy}{x^2+y^2}\hat{i} + \frac{Cx}{x^2+y^2}\hat{j}$$

$$\mathrm{div}\vec{V} = \nabla \bullet \vec{V} = \frac{\partial}{\partial x}\left(-\frac{Cy}{x^2+y^2}\right) + \frac{\partial}{\partial y}\left(\frac{Cx}{x^2+y^2}\right) = 0$$

$$\mathrm{curl}\vec{V} = \nabla \times \vec{V} = \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\hat{k}$$

$$= \left(\frac{\partial}{\partial x}\left(\frac{Cx}{x^2+y^2}\right) - \frac{\partial}{\partial y}\left(\frac{-Cy}{x^2+y^2}\right)\right)\hat{k} = 0$$

Vortex (3)

- **Free Vortex (Potential Vortex)**
  - **Cylindrical coordinates**

$$\vec{V} = \frac{C}{r}\hat{e}_\theta$$

$$\mathrm{div}\vec{V} = \nabla \bullet \vec{V} = \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{C}{r}\right) + \frac{0}{r} = 0$$

$$\boxed{e_{\theta\theta} = \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{V_r}{r}}$$

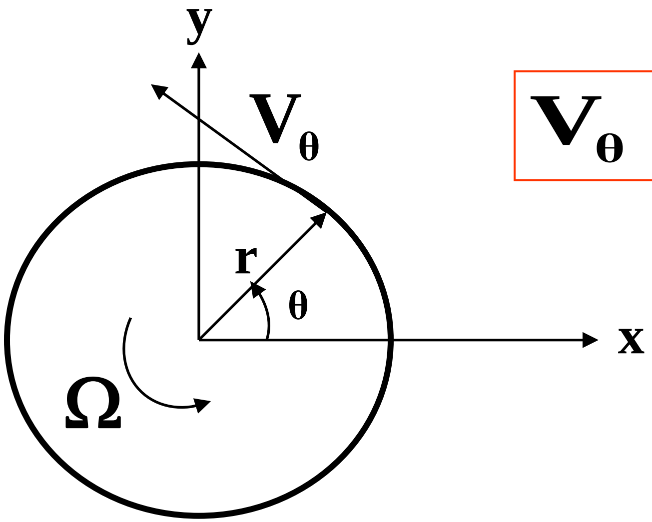
$$\boxed{\omega_z = \omega_{\theta r} = -\omega_{r\theta} = \frac{1}{2}\left\{\frac{1}{r}\frac{\partial (rV_\theta)}{\partial r} - \frac{1}{r}\frac{\partial V_r}{\partial \theta}\right\}}$$

$$\mathrm{curl}\vec{V} = \nabla \times \vec{V} = \left\{\frac{1}{r}\frac{\partial (rV_\theta)}{\partial r} - \frac{1}{r}\frac{\partial V_r}{\partial \theta}\right\}\hat{k}$$

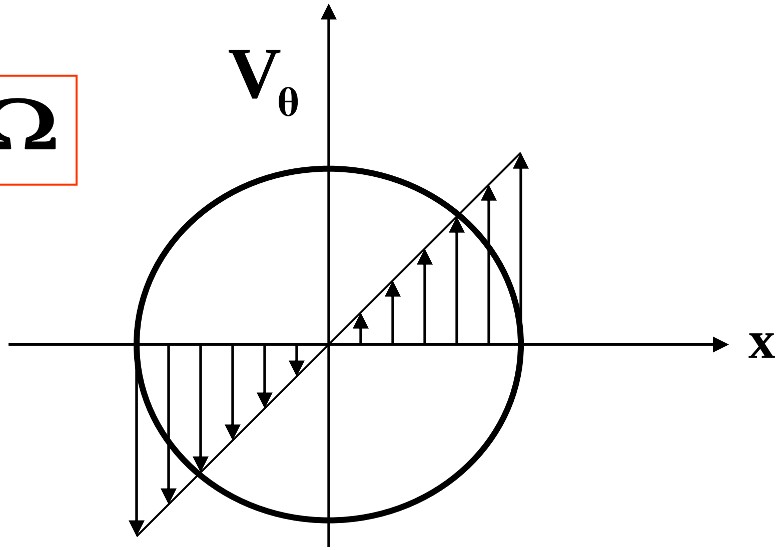
$$= \left\{\frac{1}{r}\frac{\partial\left(r\frac{C}{r}\right)}{\partial r} - \frac{1}{r}\frac{\partial (0)}{\partial \theta}\right\}\hat{k} = 0$$

Vortex (4)

Forced Vortex (Solid-Body Rotation)



$$\boxed{V_\theta = r\Omega}$$



- Cartesian coordinates: $\boxed{\vec{V} = -\Omega y\hat{i} + \Omega x\hat{j}}$

- Cylindrical coordinates: $\boxed{\vec{V} = r\Omega\hat{e}_\theta}$

Vortex (5)

- **Forced Vortex (Solid-Body Rotation)**
  - **Cartesian coordinates**

$$\vec{V} = -\Omega y\hat{i} + \Omega x\hat{j}$$

$$\mathrm{div}\vec{V} = \nabla \bullet \vec{V} = \frac{\partial}{\partial x}(-\Omega y) + \frac{\partial}{\partial y}(\Omega x) = 0$$

$$\mathrm{curl}\vec{V} = \nabla \times \vec{V} = \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\hat{k}$$

$$= \left(\frac{\partial(\Omega x)}{\partial x} - \frac{\partial(-\Omega y)}{\partial y}\right)\hat{k} = 2\Omega\hat{k}$$

Note: **For a forced vortex (solid-body rotation), the curl of velocity at a point is twice the rotation vector.**

Vortex (6)

Forced Vortex (Solid-Body Rotation)

- Cylindrical coordinates

$$\vec{V} = r\Omega\,\hat{e}_\theta$$

$$e_{\theta\theta} = \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{V_r}{r}$$

$$\mathrm{div}\vec{V} = \nabla \bullet \vec{V} = \frac{1}{r}\frac{\partial}{\partial \theta}(r\Omega) + \frac{0}{r} = 0$$

$$\omega_z = \omega_{\theta r} = -\omega_{r\theta} = \frac{1}{2}\left\{\frac{1}{r}\frac{\partial (rV_\theta)}{\partial r} - \frac{1}{r}\frac{\partial V_r}{\partial \theta}\right\}$$

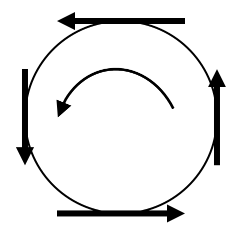
$$\mathrm{curl}\vec{V} = \nabla \times \vec{V} = \left\{\frac{1}{r}\frac{\partial (rV_\theta)}{\partial r} - \frac{1}{r}\frac{\partial V_r}{\partial \theta}\right\}\hat{k}$$

$$= \left\{\frac{1}{r}\frac{\partial (r^2\Omega)}{\partial r} - \frac{1}{r}\frac{\partial (0)}{\partial \theta}\right\}\hat{k} = 2\Omega\hat{k}$$

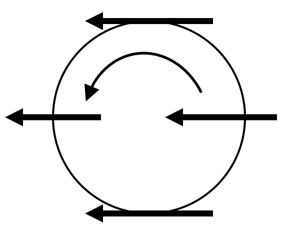
Vortex (7)

- Angular velocity and vorticity are constant in a forced vortex (**solid-body rotation**).
- The free vortex (**potential vortex**) has zero vorticity outside the core, and its angular momentum remains constant.

Movement of a tiny rod floating in water:



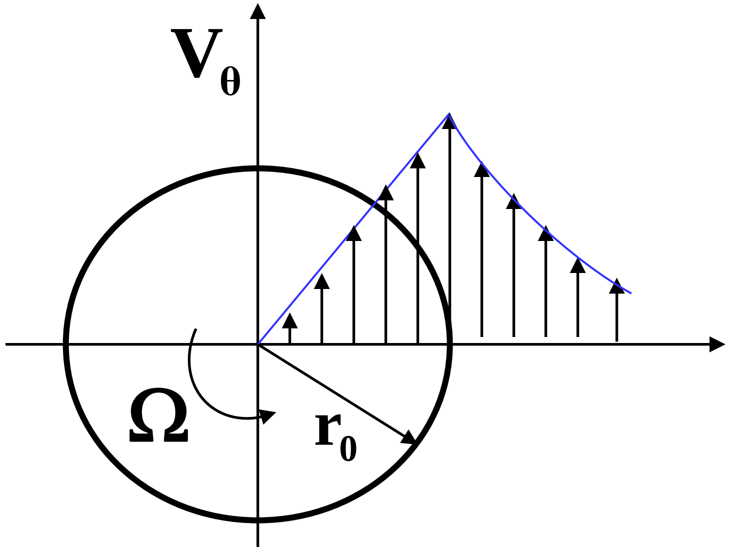
Forced Vortex



Free Vortex

Vortex (8)

- **Rankine Vortex (Combination of Free and Forced Vortices)**



$$V_\theta(r) = \begin{cases} r\,\Omega, & (r \le r_o) \quad \text{Forced Vortex} \\ \dfrac{\Omega\, r_o^2}{r}, & (r > r_o) \quad \text{Free Vortex} \end{cases}$$

Vortex (9)

A Generalized Vortex

- A generalized vortex is characterized by an arbitrary radial distribution of the swirl factor

$$S_f = f_1(r)$$

where the swirl factor is given by the equation: $S_f = \dfrac{V_\theta}{r\Omega}$

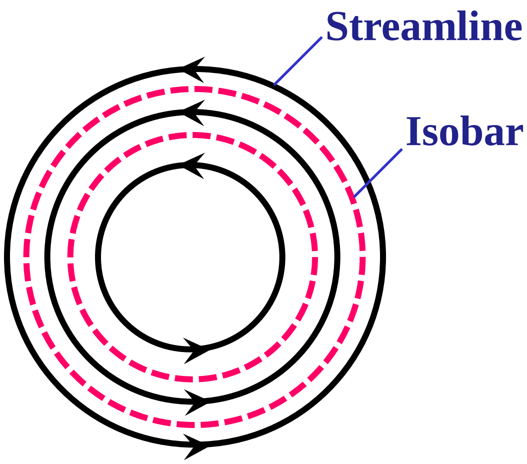
- A nonisentropic generalized vortex additionally features an arbitrary radial variation of its total temperature given by

$$T_t = f_2(r)$$

Vortex (10)

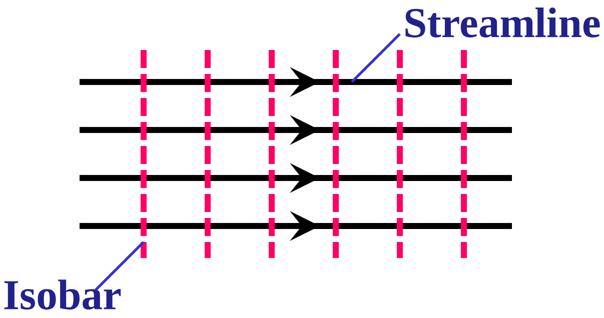
Vortex versus Parallel Flows

- **Vortex Flow**



Streamlines and isobars are concentric circles.

- **Parallel Flow**



Streamlines and isobars are normal to each other.

Vortex (11)

- A vortex is characterized by swirl or tangential velocity at any point in a flow field, and it is generally specified by a swirl factor (S_f)

$$\boxed{S_f = \frac{\text{Fluid RPM}}{\text{Rotor RPM}}} \qquad \boxed{XK \equiv S_f}$$

- Free Vortex

$$rV_\theta = \text{Constant} \quad \text{or} \quad S_f = \frac{C_1}{r^2}$$

- Forced Vortex (Solid-Body Rotation)

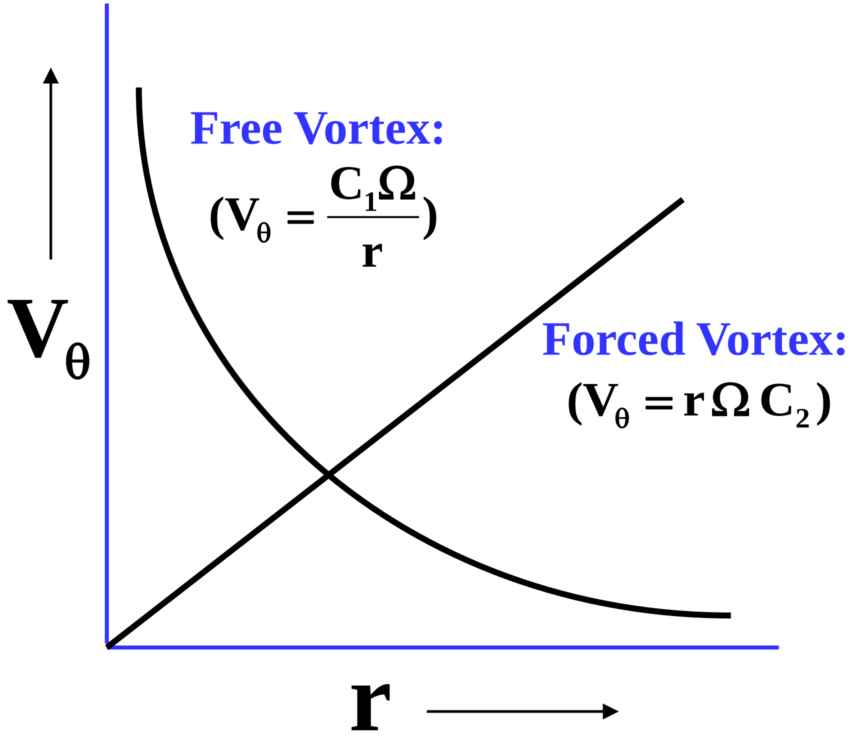
$$S_f = C_2$$

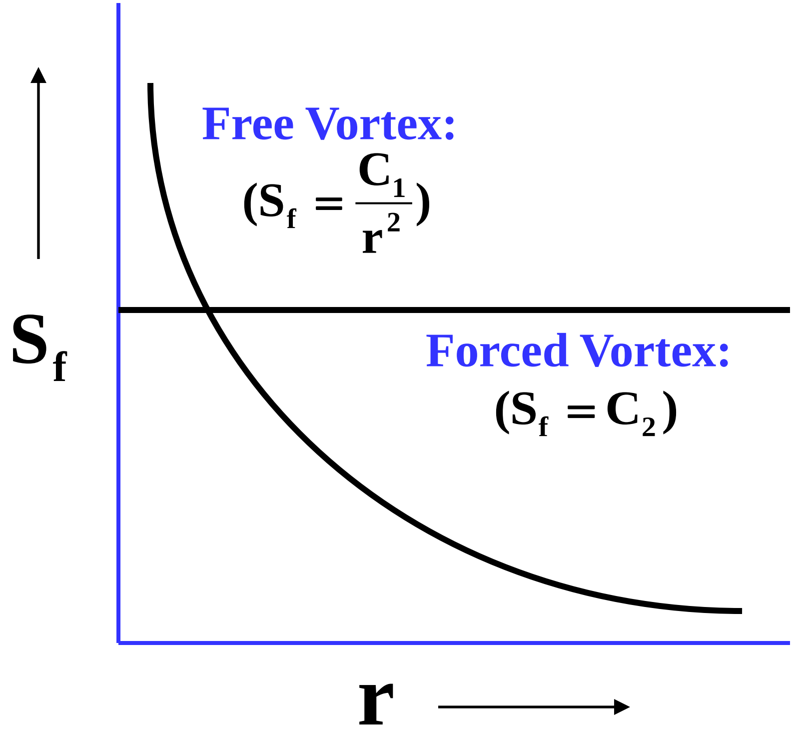
- Generalized Vortex

$$S_f = f(r)$$

Vortex (12)

- Free Vortex Versus Forced Vortex





Static Pressure and Static Temperature Changes in an Isentropic Free Vortex (1)

- **For an isentropic process:**

$$\frac{P_s}{\rho_s^{\kappa}} = C$$

Taking log of the above equation yields

$$\ln P_s = \ln C + \kappa \ln \rho_s = \ln C + \kappa \ln\left(\frac{P_s}{RT_s}\right)$$

$$= \ln C + \kappa \ln P_s - \kappa \ln T_s - \kappa \ln R$$

Differentiating the above equation and rearranging terms yield

$$\boxed{\frac{dP_s}{P_s} = \left(\frac{\kappa}{\kappa - 1}\right)\frac{dT_s}{T_s}} \qquad (1)$$

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Static Pressure and Static Temperature Changes in an Isentropic Free Vortex (2)

- Radial pressure gradient in a vortex is given by

$$\frac{dP_s}{dr} = \frac{\rho_s V_\theta^2}{r} = \frac{P_s}{RT_s}\frac{V_\theta^2}{r}$$

$$\boxed{\frac{dP_s}{P_s} = \frac{V_\theta^2}{RT}\frac{dr}{r}} \qquad (2)$$

Substituting for $\dfrac{dP_s}{P_s}$ from Eq. (2) in Eq. (1) yields

$$\boxed{dT_s = \left(\frac{\kappa - 1}{\kappa R}\right)\frac{V_\theta^2 dr}{r} = \frac{V_\theta^2}{c_p}\frac{dr}{r}} \qquad (3)$$

Static Pressure and Static Temperature Changes in an Isentropic Free Vortex (3)

- For a free vortex

$$\boxed{V_\theta = \frac{C_3}{r}} \quad \text{or} \quad S_f = \frac{C_1}{r^2} \qquad (4)$$

where

$$C_1 = \frac{C_3}{\Omega}$$

Substituting for V_θ from Eq. (4) into Eq. (3) yields

$$\int_1^2 dT_s = \left(\frac{C_3^2}{C_p}\right)\int_1^2 \frac{dr}{r^3}$$

Static Pressure and Static Temperature Changes in an Isentropic Free Vortex (4)

$$\left(T_{s_2} - T_{s_1}\right) = \left(\frac{C_3^2}{2c_p}\right)\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)$$

Using $\left(\frac{\kappa - 1}{\kappa R}\right) = \frac{1}{c_p}$, we obtain

$$\left(T_{s_2} - T_{s_1}\right) = \left(\frac{\kappa - 1}{2}\right)\left(\frac{C_3^2}{\kappa R}\right)\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)$$

Static Pressure and Static Temperature Changes in an Isentropic Free Vortex (5)

$$\frac{T_{s_2}}{T_{s_1}} = 1 + \left(\frac{\kappa - 1}{2}\right) M_{\theta_1}^2 \left(1 - \frac{r_1^2}{r_2^2}\right)$$

$$\boxed{\frac{T_{s_2}}{T_{s_1}} = 1 + \left(\frac{\kappa - 1}{2}\right) M_{\theta_1}^2 \left(\frac{r_2^2}{r_1^2} - 1\right) \frac{r_1^2}{r_2^2}} \qquad (5)$$

where

$$M_{\theta_1} = \frac{V_{\theta_1}}{\sqrt{\kappa R T_{s_1}}} = \frac{C_3}{\left(\sqrt{\kappa R T_{s_1}}\right) r_1}$$

Static Pressure and Static Temperature Changes in an Isentropic Free Vortex (6)

- Pressure ratio between any two points in a free vortex is then computed using the isentropic relation

$$\frac{P_{s_2}}{P_{s_1}} = \left(\frac{T_{s_2}}{T_{s_1}}\right)^{\frac{\kappa}{\kappa-1}} = \left[1+\left(\frac{\kappa-1}{2}\right)M_{\theta_1}^2\left(1-\frac{r_1^2}{r_2^2}\right)\right]^{\frac{\kappa}{\kappa-1}}$$

$$\boxed{\frac{P_{s_2}}{P_{s_1}} = \left(\frac{T_{s2}}{T_{s1}}\right)^{\frac{\kappa}{\kappa-1}} = \left[1+\left(\frac{\kappa-1}{2}\right)M_{\theta_1}^2\left(\frac{r_2^2}{r_1^2}-1\right)\frac{r_1^2}{r_2^2}\right]^{\frac{\kappa}{\kappa-1}} \qquad (6)}$$

Static Pressure and Static Temperature Changes in an Isentropic Forced Vortex (1)

- For a forced vortex

$$V_\theta = r\, S_f\, \Omega \qquad (7)$$

Substituting for V_θ from Eq. (7) into Eq. (3) and integrating from 1 to 2 yields

$$\int_1^2 dT_s = \left(\frac{S_f^{\,2}\Omega^2}{c_p}\right)\int_1^2 r\,dr$$

$$(T_{s_2} - T_{s_1}) = \left(\frac{S_f^{\,2}\Omega^2}{2c_p}\right)(r_2^2 - r_1^2)$$

Static Pressure and Static Temperature Changes in an Isentropic Forced Vortex (2)

Again using $\left(\dfrac{\kappa-1}{\kappa R}\right)=\dfrac{1}{c_p}$, we obtain

$$(T_{s_2}-T_{s_1})=\left(\frac{\kappa-1}{2}\right)\left(\frac{S_f^{\,2}\Omega^2}{\kappa R}\right)(r_2^2-r_1^2)$$

$$\boxed{\frac{T_{s_2}}{T_{s_1}}=1+\left(\frac{\kappa-1}{2}\right)M_{\theta_1}^2\left(\frac{r_2^2}{r_1^2}-1\right)} \qquad (8)$$

Static Pressure and Static Temperature Changes in an Isentropic Forced Vortex (3)

where

$$M_{\theta_1} = \frac{r_1 S_f \Omega}{\sqrt{\kappa R T_{s_1}}}$$

- Pressure ratio between any two points in a forced vortex is then computed using following the isentropic relation

$$\frac{P_{s_2}}{P_{s_1}} = \left(\frac{T_{s_2}}{T_{s_1}}\right)^{\frac{\kappa}{\kappa-1}} = \left(1 + \left(\frac{\kappa-1}{2}\right) M_{\theta_1}^2 \left(\frac{r_2^2}{r_1^2} - 1\right)\right)^{\frac{\kappa}{\kappa-1}} \qquad (9)$$

Free Vortex versus Forced Vortex (1)

- In a free vortex, angular momentum, not angular velocity, remains constant.
- In a forced vortex, angular velocity, not angular momentum, remains constant.
- Note that from Equations (5) and (8), we obtain

$$\left(\frac{T_{s_2}-T_{s_1}}{T_{s_1}}\right)_{\text{Forced Vortex}} = \left(\frac{T_{s_2}-T_{s_1}}{T_{s_1}}\right)_{\text{Free Vortex}} \left(\frac{r_2^2}{r_1^2}\right)$$

$$\left(\frac{T_{s_{out}}-T_{s_{in}}}{T_{s_{in}}}\right)_{\text{Forced Vortex}} = \left(\frac{T_{s_{out}}-T_{s_{in}}}{T_{s_{in}}}\right)_{\text{Free Vortex}} \left(\frac{r_{out}^2}{r_{in}^2}\right) \qquad (10)$$

In Eq. (10), subscripts "in" and "out" refer to inlet and outlet, respectively

Free Vortex versus Forced Vortex (2)

- **From Eq. (10), note that for both radially outward and inward flows with identical inlet conditions, outlet static temperature for a forced vortex is always higher than that for a free vortex.**
- **Similarly, for both radially outward and inward flows with identical inlet conditions, outlet static pressure for a forced vortex is always higher than that for a free vortex.**
- **It's a simple exercise to show that the total temperature and total pressure of a free vortex in an inertial reference frame remain constant.** ($T_{t_2} = T_{t_1}$ **and** $P_{t_2} = P_{t_1}$)

For an isentropic forced vortex:

$$\frac{T_{t_2}}{T_{t_1}} = 1 + \left(\frac{\Omega^2 (r_2^2 - r_1^2)}{c_P T_{t_1}} \right)$$

and

$$\frac{P_{t_2}}{P_{t_1}} = \left(\frac{T_{t_2}}{T_{t_1}} \right)^{\frac{\kappa}{\kappa-1}} = \left[1 + \left(\frac{\Omega^2 (r_2^2 - r_1^2)}{c_P T_{t_1}} \right) \right]^{\frac{\kappa}{\kappa-1}}$$

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Free Vortex versus Forced Vortex (3)

- A joint equation for both **free vortex** and **force vortex** is written as

$$\boxed{V_\theta = \frac{C_4}{r^n}}$$

where $n = 1$ for a **free vortex**, and $n = -1$ for a **forced vortex**.

- Static Temperature Change:

$$\frac{T_{s_2}}{T_{s_1}} = 1 + \left(\frac{\kappa - 1}{2n}\right) M_{\theta_1}^2 \left\{1 - \left(\frac{r_1}{r_2}\right)^{2n}\right\}$$

- Static Pressure Change:

$$\frac{P_{s_2}}{P_{s_1}} = \left[1 + \left(\frac{\kappa - 1}{2n}\right) M_{\theta_1}^2 \left\{1 - \left(\frac{r_1}{r_2}\right)^{2n}\right\}\right]^{\frac{\kappa}{\kappa - 1}}$$

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Free Vortex versus Forced Vortex (4)

- **Pressure and Temperature Changes in an Isentropic Free Vortex**

$$\frac{T_{s_2}}{T_{s_1}} = 1 + \left(\frac{\kappa - 1}{2}\right) M_{\theta_1}^2 \left(\frac{r_2^2}{r_1^2} - 1\right)\frac{r_1^2}{r_2^2}$$

$$M_{\theta_1} = \frac{V_{\theta_1}}{\sqrt{\kappa R T_{s_1}}} = \frac{C_3}{\left(\sqrt{\kappa R T_{s_1}}\right) r_1}$$

$$\frac{P_{s_2}}{P_{s_1}} = \left(\frac{T_{s_2}}{T_{s_1}}\right)^{\frac{\kappa}{\kappa-1}} = \left[1 + \left(\frac{\kappa - 1}{2}\right) M_{\theta_1}^2 \left(\frac{r_2^2}{r_1^2} - 1\right)\frac{r_1^2}{r_2^2}\right]^{\frac{\kappa}{\kappa-1}}$$

$$T_{t_2} = T_{t_1}$$

$$P_{t_2} = P_{t_1}$$

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Free Vortex versus Forced Vortex (5)

- Pressure and Temperature Changes in an Isentropic Forced Vortex

$$\frac{T_{s_2}}{T_{s_1}} = 1 + \left(\frac{\kappa - 1}{2}\right) M_{\theta_1}^2 \left(\frac{r_2^2}{r_1^2} - 1\right)$$

$$M_{\theta_1} = \frac{r_1 S_f \Omega}{\sqrt{\kappa R T_{s_1}}}$$

$$\frac{P_{s_2}}{P_{s_1}} = \left(\frac{T_{s_2}}{T_{s_1}}\right)^{\frac{\kappa}{\kappa-1}} = \left(1 + \left(\frac{\kappa - 1}{2}\right) M_{\theta_1}^2 \left(\frac{r_2^2}{r_1^2} - 1\right)\right)^{\frac{\kappa}{\kappa-1}}$$

$$T_{t_2} = T_{t_1} + \frac{V_{\theta_2}^2 - V_{\theta_1}^2}{c_P} = T_{t_1} + \frac{S_f^2 \Omega^2 (r_2^2 - r_1^2)}{c_P}$$

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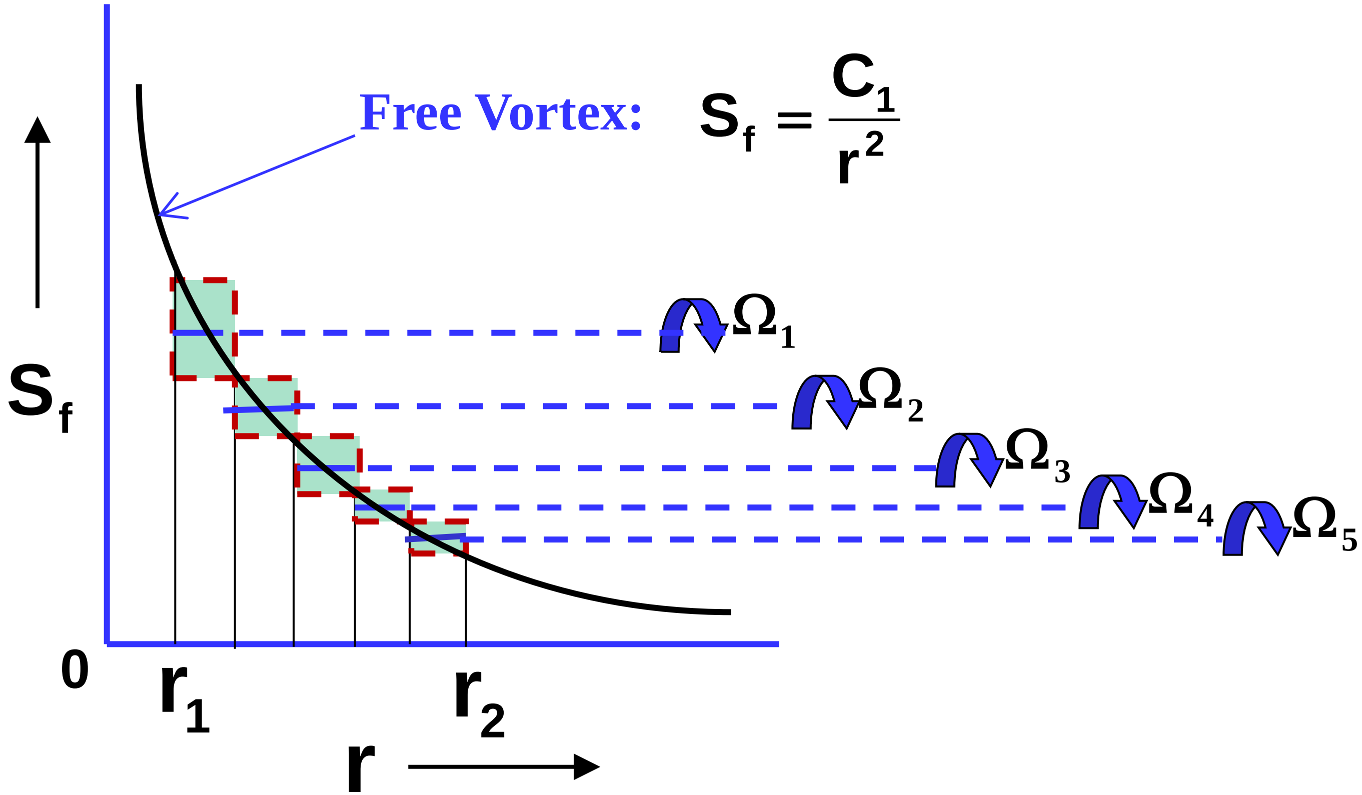
Free Vortex versus Forced Vortex (6)

Forced Vortex Modeling of a Free Vortex



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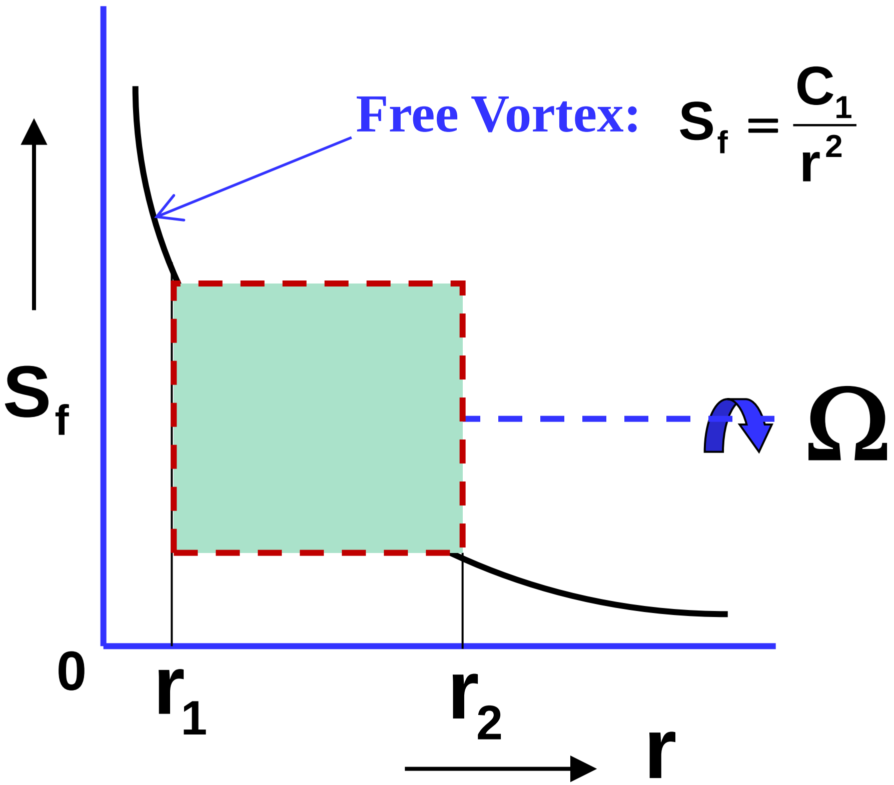
Free Vortex versus Forced Vortex (7)

Forced Vortex Modeling of a Free Vortex



- **For the free vortex:**
  - At r_1 : $V_{\theta_1}, \ P_{s_1} \ T_{s_1}$
  - At r_2 : $V_{\theta_2}, \ P_{s_2} \ T_{s_2}$

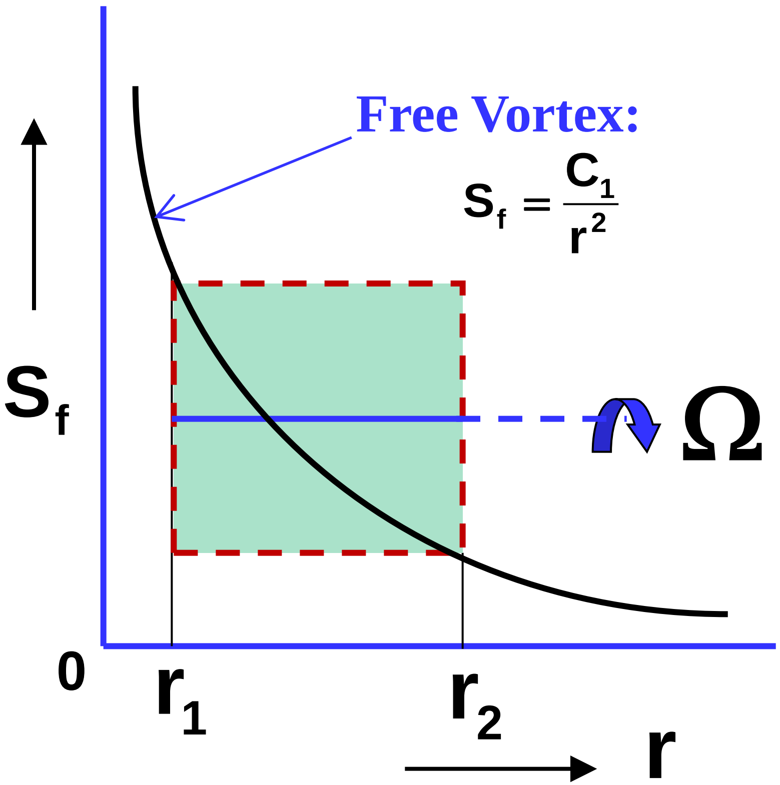
$$\boxed{V_{\theta_2} = \frac{V_{\theta_1} r_1}{r_2}}$$

- **For the equivalent forced vortex:**

$$\boxed{\Omega = \frac{V_{\theta_1} + V_{\theta_2}}{r_1 + r_2} = \frac{V_{\theta_1}}{r_2}}$$

Free Vortex versus Forced Vortex (8)

❑ Forced Vortex Modeling of a Free Vortex



❖ For the free vortex, we can write

$$\boxed{\left(\frac{T_{s_2}}{T_{s_1}}\right)_{\text{free vortex}} = 1 + \left(\frac{\kappa - 1}{2}\right) M_{\theta_1}^2 \left(\frac{r_2^2}{r_1^2} - 1\right) \frac{r_1^2}{r_2^2}}$$

$$\boxed{M_{\theta_1} = \frac{V_{\theta_1}}{\sqrt{\kappa R T_{s_1}}}}$$

$$\boxed{\left(\frac{T_{s_2}}{T_{s_1}}\right)_{\text{free vortex}} = 1 + \left(\frac{\kappa - 1}{2}\right) \frac{V_{\theta 1}^2}{\kappa R T_{s_1}} \left(\frac{r_2^2}{r_1^2} - 1\right) \frac{r_1^2}{r_2^2}}$$

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Free Vortex versus Forced Vortex (9)

Forced Vortex Modeling of a Free Vortex

- For the equivalent forced vortex, we can write

$$\left(\frac{T_{s_2}}{T_{s_1}}\right)_{\text{forced vortex}} = 1 + \left(\frac{\kappa - 1}{2}\right) M_{\theta_1}^2 \left(\frac{r_2^2}{r_1^2} - 1\right)$$

$$M_{\theta_1} = \frac{r_1 \Omega}{\sqrt{\kappa R T_{s_1}}}$$

$$\left(\frac{T_{s_2}}{T_{s_1}}\right)_{\text{forced vortex}} = 1 + \left(\frac{\kappa - 1}{2}\right) \frac{r_1^2 \Omega^2}{\kappa R T_{s_1}} \left(\frac{r_2^2}{r_1^2} - 1\right)$$

$$\left(\frac{T_{s_2}}{T_{s_1}}\right)_{\text{forced vortex}} = 1 + \left(\frac{\kappa - 1}{2}\right) \frac{V_{\theta_1}^2}{\kappa R T_{s_1}} \frac{r_1^2}{r_2^2} \left(\frac{r_2^2}{r_1^2} - 1\right) = \left(\frac{T_{s_2}}{T_{s_1}}\right)_{\text{free vortex}}$$

Free Vortex versus Forced Vortex (10)

❑ Forced Vortex Modeling of a Free Vortex

Since for forced vortex core with the mean angular velocity, we have shown that

$$\boxed{\left(\frac{T_{s_2}}{T_{s_1}}\right)_{\text{forced vortex}} = \left(\frac{T_{s_2}}{T_{s_1}}\right)_{\text{free vortex}}}$$

We can also write

$$\boxed{\left(\frac{P_{s_2}}{P_{s_1}}\right)_{\text{forced vortex}} = \left(\frac{T_{s_2}}{T_{s_1}}\right)_{\text{forced vortex}}^{\frac{\kappa}{\kappa-1}} = \left(\frac{P_{s_2}}{P_{s_1}}\right)_{\text{free vortex}} = \left(\frac{T_{s_2}}{T_{s_1}}\right)_{\text{free vortex}}^{\frac{\kappa}{\kappa-1}}}$$

Isothermal Forced Vortex (1)

- **Radial Momentum Equation:** $$\frac{dP_s}{dr} = \frac{\rho V_\theta^2}{r}$$

For a forced vortex: $V_\theta = rS_f\Omega$

For an isothermal vortex ($T_s =$ constant): $$\rho = \frac{P_s}{RT_s}$$

Substituting in the radial equilibrium equation yields

$$\frac{dP_s}{P_s} = \frac{r^2 S_f^2 \Omega^2}{RT_s}\frac{dr}{r} = \left(\frac{S_f^2 \Omega^2}{RT_s}\right) r\,dr$$

Isothermal Forced Vortex (2)

$$\int_{P_{s1}}^{P_{s2}} \frac{dP_s}{P_s} = \left(\frac{S_f^2 \Omega^2}{R T_s} \right) \int_{r_1}^{r_2} r\, dr$$

$$\ln\left(\frac{P_{s_2}}{P_{s_1}} \right) = \left(\frac{r_1^2 S_f^2 \Omega^2}{2 R T_s} \right) \left(\frac{r_2^2}{r_1^2} - 1 \right)$$

$$\boxed{\ln\left(\frac{P_{s_2}}{P_{s_1}} \right) = \frac{\kappa M_{\theta_1}^2}{2} \left(\frac{r_2^2}{r_1^2} - 1 \right)}$$

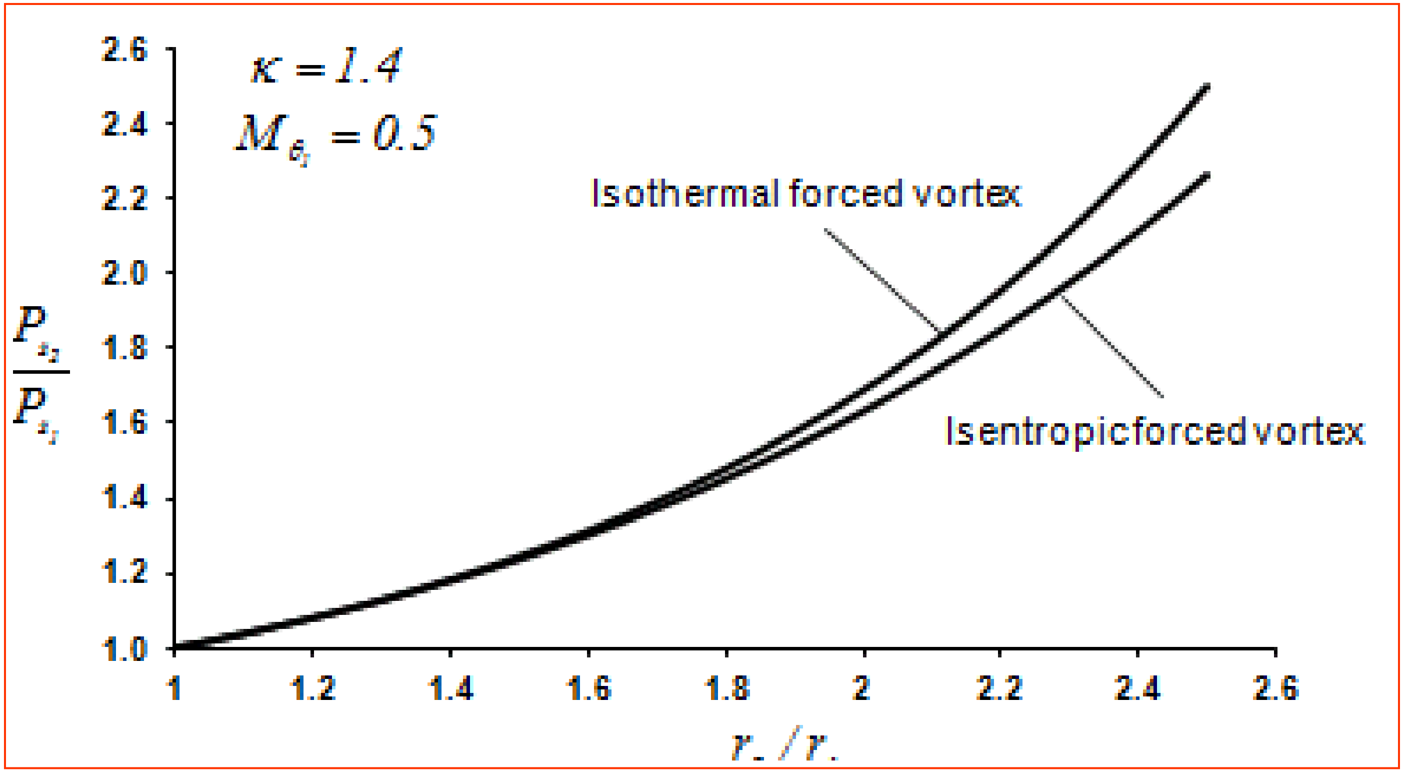
where

$$\boxed{M_{\theta_1} = \frac{r_1 S_f \Omega}{\sqrt{\kappa R T_{s_1}}}}$$

$$\boxed{T_{s_1} = T_{s_2} = T_s}$$

Isothermal Forced Vortex (3)

- Comparison of Isentropic and Isothermal Pressure Rises in a Forced Vortex



Pressure and Temperature Changes in a Nonisentropic Generalized Vortex (1)

- Radial pressure gradient in a vortex is given by

$$\frac{dP_s}{dr} = \frac{\rho V_\theta^2}{r} \quad \Longrightarrow \quad \frac{dP_s}{P_s} = \frac{V_\theta^2}{RT_s}\frac{dr}{r}$$

which can be expressed in terms of rotational Mach number as

$$\frac{dP_s}{P_s} = \kappa\left(\frac{M_\theta^2}{r}\right)dr$$

where

$$M_\theta = \frac{V_\theta}{\sqrt{\kappa R T_s}}$$

Pressure and Temperature Changes in a Nonisentropic Generalized Vortex (2)

$$V_\theta = S_f\,\Omega\,r = f_1\,\Omega\,r \quad \text{and} \quad T_s = T_t - \frac{V_\theta^2}{2c_p} = f_2 - \frac{f_1^2\,\Omega^2\,r^2}{2c_p}$$

Thus, integration between r_1 and r_2 yields

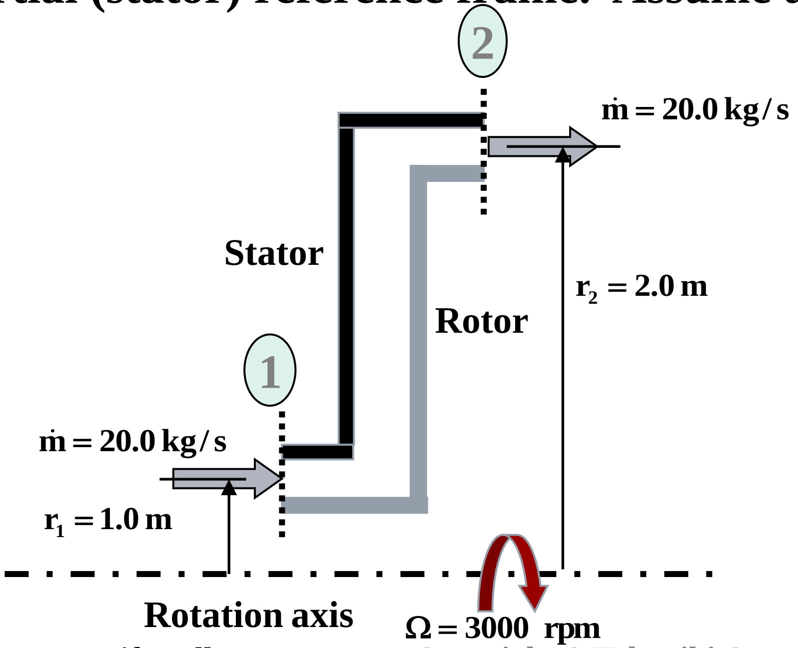
$$\int_{r_1}^{r_2} \frac{dP_s}{P_s} = \kappa \int_{r_1}^{r_2} \left(\frac{M_\theta^2}{r} \right) dr \Longrightarrow \ln\left(\frac{P_{s_2}}{P_{s_1}} \right) = \kappa G$$

$$\boxed{P_{s_2} = P_{s_1} e^{\kappa G}}$$

where G is computed numerically using, for example, the Simpson's one-third rule.

A Typical Windage Problem (1)

A rotor-stator cavity of a 50-Hertz (3000 rpm) gas turbine engine is shown below. Coolant air at 400 ^{O}C (absolute total), swirling at 60% of the rotor rpm, enters the cavity at the inner radius. It exits the cavity at the outer radius with a swirl of 40% of the rotor rpm. The mass flow rate of the coolant air is 20 kg/s. If the total frictional torque from the stator surface acting on the cavity air is 10 Nm, **find the exit total temperature of this air**. The rotor-stator surfaces are adiabatic (zero heat transfer). All quantities are given in the inertial (stator) reference frame. Assume air with c_P = 1067 J/(kg.K).



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A Typical Windage Problem (2)

Solution

- Angular velocity of the rotor:

$$\Omega = \frac{3000 \times 2\pi}{60} = 314.16 \quad \text{rad/s}$$

- Air tangential velocity at inlet (Section 1):

$$V_{\theta_1} = 0.6 \times 314.16 \times 1.0 = 188.50 \quad \text{m/s}$$

- Air tangential velocity at outlet (Section 2):

$$V_{\theta_2} = 0.4 \times 314.16 \times 2.0 = 251.33 \quad \text{m/s}$$

A Typical Windage Problem (3)

- Torque and angular momentum balance over the control volume between Sections 1 and 2 yields

$$\Gamma_{rotor} - \Gamma_{stator} = \dot{m}(r_2 V_{\theta_2} - r_1 V_{\theta_1})$$

$$\Gamma_{rotor} = \Gamma_{stator} + \dot{m}(r_2 V_{\theta_2} - r_1 V_{\theta_1})$$

$$= 10 + 20(2.0 \times 251.33 - 1.0 \times 188.50)$$

$$= 6293.18 \text{ Nm}$$

- Air temperature increase in the rotor cavity is due to work transfer from the rotor

$$\Delta T_t = \frac{\Gamma_{rotor} \times \Omega}{\dot{m} c_P} = \frac{6293.18 \times 314.16}{20 \times 1067} = 92.6 \text{ K}$$

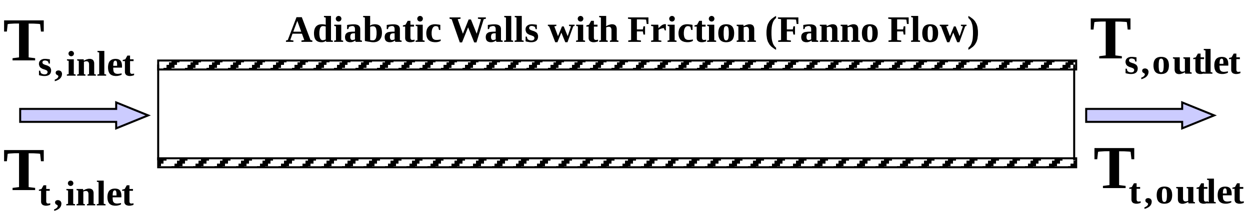
Hence exit total temperature of air = 400 + 92.6 = 492.6 ^{O}C

Windage (1)

What is Windage?

Windage $\neq$ Viscous Dissipation

- No Windage in a Stationary Channel



$T_{s,outlet} < T_{s,inlet}$ $\qquad$ $T_{t,outlet} = T_{t,inlet}$

- Windage power is the net rotor power input into the surrounding fluid.

Windage (2)

Pumping Power into a Forced Vortex

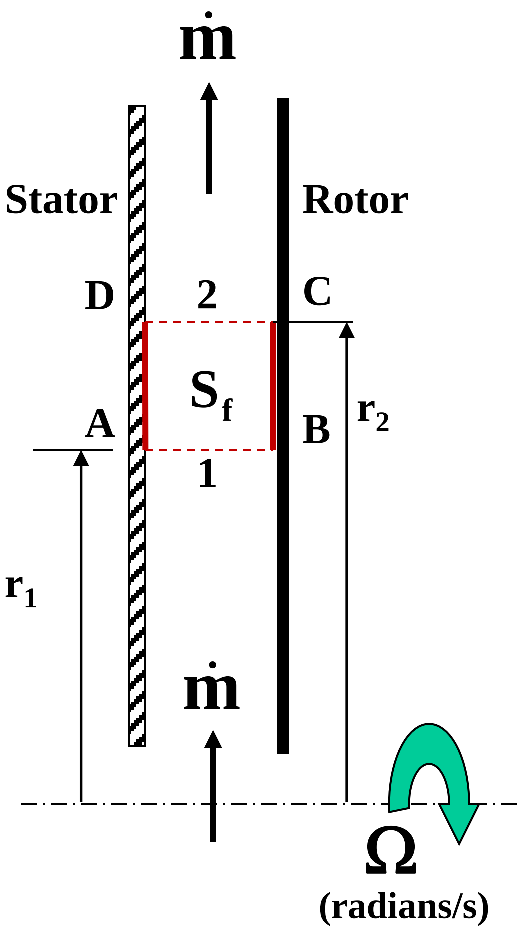
- Constant Flow Energy Equation for the Control Volume ABCD yields

$$\dot{m}(h_{t_2} - h_{t_1}) = W_{pumping} + Q$$

- Assume a Forced Vortex of Constant Swirl Factor (S_f) Spanning from r_1 to r_2

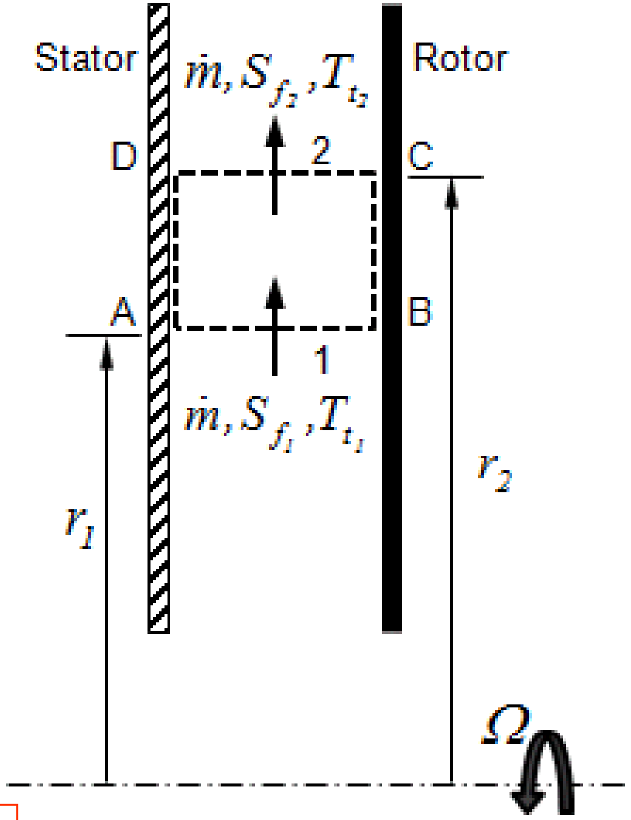
With $Q = 0$

$$\boxed{W_{pumping} = \dot{m}\,(S_f\,\Omega)^2(r_2^2 - r_1^2)}$$



Windage (3)

Consider Control Volume ABCD



- **Torque/Angular Momentum Balance**

$$\Gamma_{BC,rotor} - \Gamma_{AD,stator} = (\dot{m}V_{\theta_2} r_2 - \dot{m}V_{\theta_1} r_1)$$

$$= \dot{m}(S_{f_2} r_2^2 - S_{f_1} r_1^2)\Omega$$

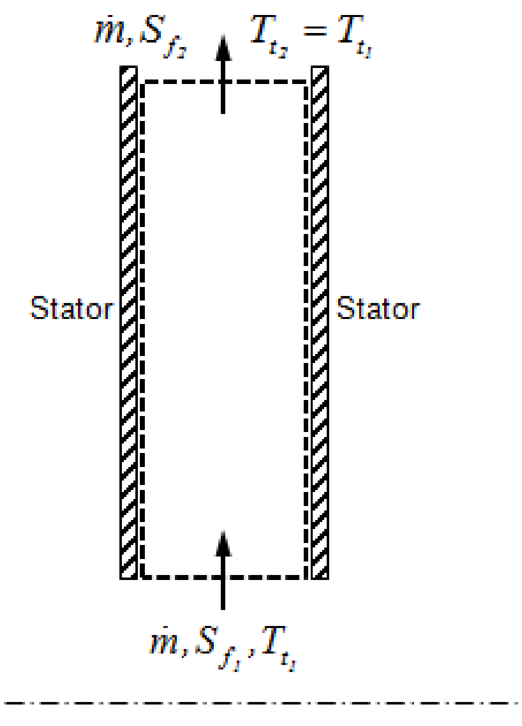
- **Windage**

Don’t Use This Equation!

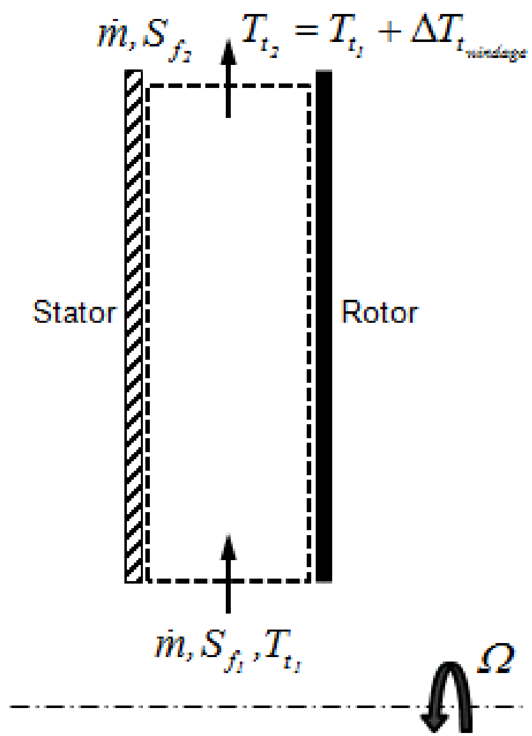
$$\boxed{W_{windage} = \Gamma_{BC,rotor}\,\Omega}$$

$$\boxed{W_{windage} = \Gamma_{AD,stator}\,\Omega + \dot{m}(S_{f_2} r_2^2 - S_{f_1} r_1^2)\Omega^2}$$

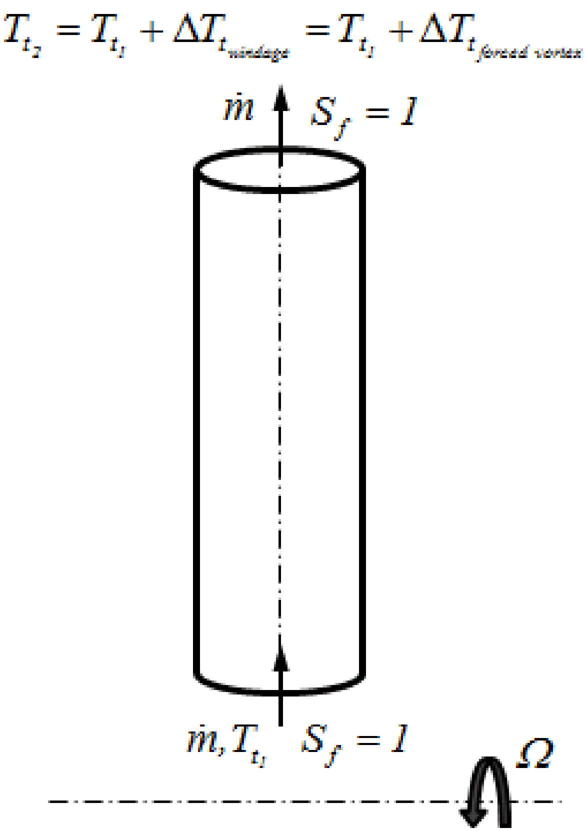
Windage (4)



All-stator cavity



Rotor-stator cavity



Rotating pipe

Compressible Mass Flow Functions (1)

Total Pressure Flow Function

$$\dot{m}_{ideal} = \frac{F_{f_t} A P_t}{\sqrt{T_t}} = \frac{\hat{F}_{f_t} A P_t}{\sqrt{R T_t}}$$

$$F_{f_t} = M \sqrt{\frac{\kappa}{R\left(1 + \frac{\kappa - 1}{2} M^2\right)^{\frac{\kappa+1}{\kappa-1}}}}$$

$$\hat{F}_{f_t} = M \sqrt{\frac{\kappa}{\left(1 + \frac{\kappa - 1}{2} M^2\right)^{\frac{\kappa+1}{\kappa-1}}}}$$

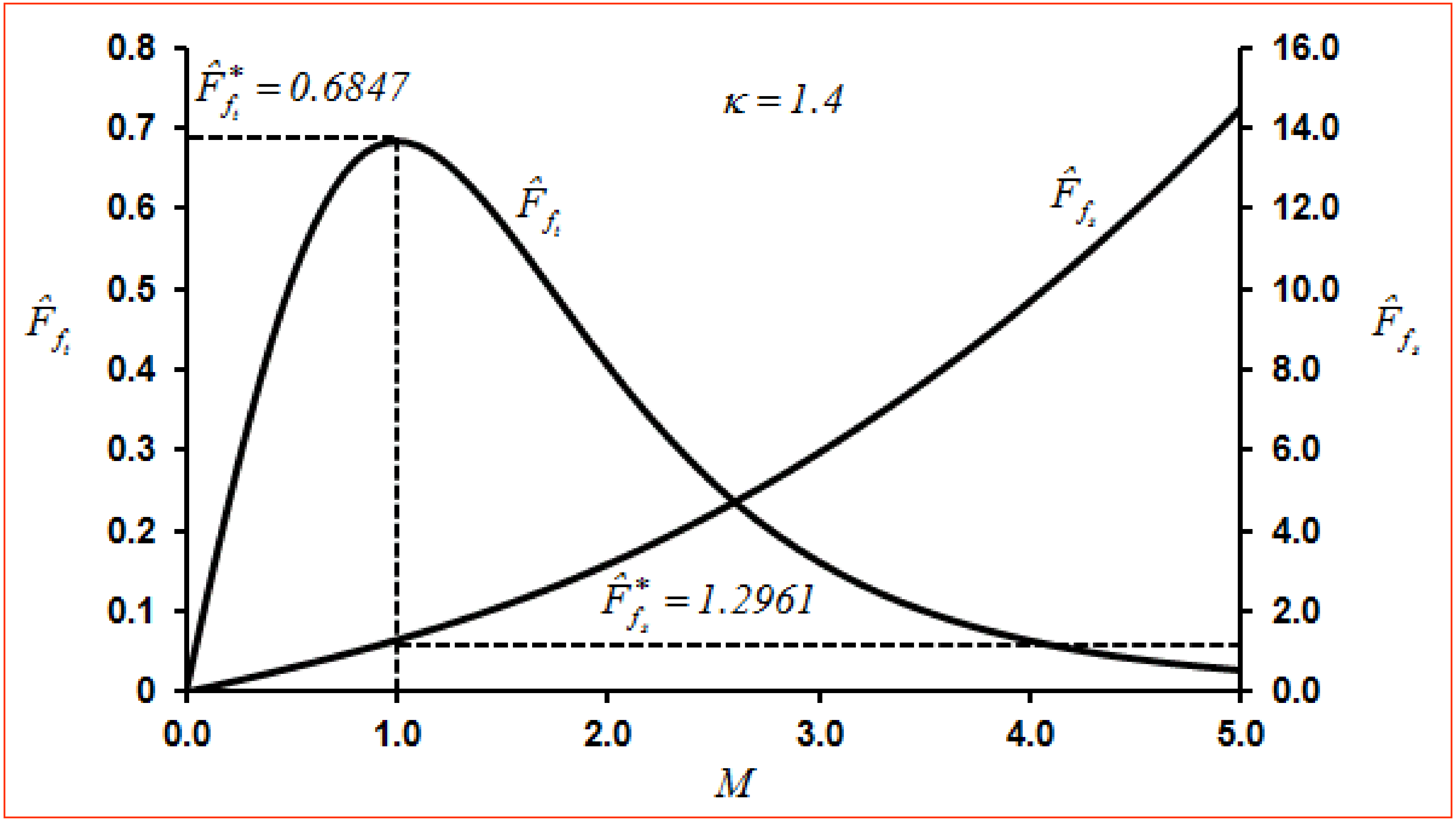
Static Pressure Flow Function

$$\dot{m}_{ideal} = \frac{F_{f_s} A P_s}{\sqrt{T_t}} = \frac{\hat{F}_{f_s} A P_s}{\sqrt{R T_t}}$$

$$F_{f_s} = M \sqrt{\frac{\kappa\left(1 + \frac{\kappa - 1}{2} M^2\right)}{R}}$$

$$\hat{F}_{f_s} = M \sqrt{\kappa\left(1 + \frac{\kappa - 1}{2} M^2\right)}$$

Compressible Mass Flow Functions (2)



Compressible Mass Flow Functions (3)

- **Finding Mach Number For a Given Static-Pressure Mass Flow Function**

$$\hat{F}_{f_s}^2 = \kappa M^2 + 0.5\kappa(\kappa - 1) M^4$$

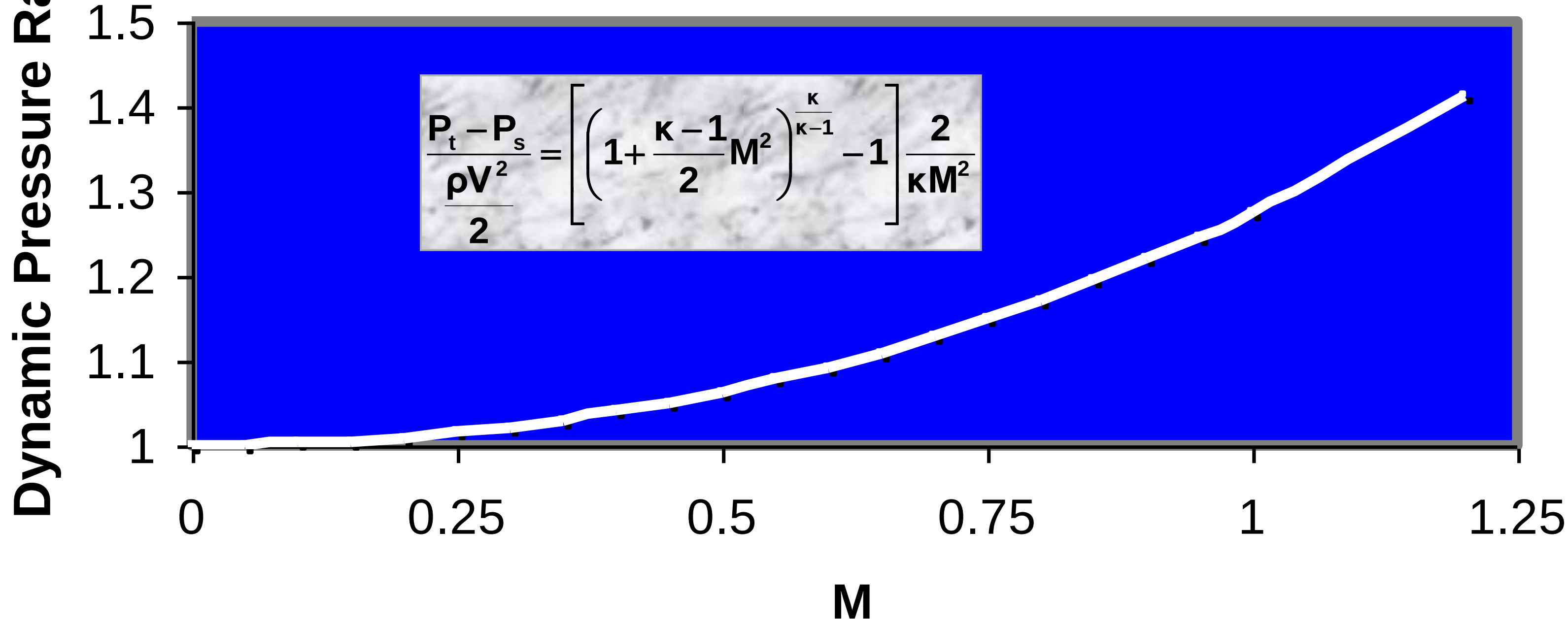
$$0.5\kappa(\kappa - 1) M^4 + \kappa M^2 - \hat{F}_{f_s}^2 = 0$$

$$\boxed{M = \left[\frac{-\kappa + \sqrt{\kappa^2 + 2\kappa(\kappa - 1)\hat{F}_{f_s}^2}}{\kappa(\kappa - 1)} \right]^{\frac{1}{2}}}$$

Dynamic Pressure

❑ Dynamic Pressure Variation in a Compressible Flow

$$\text{Dynamic Pressure Ratio} = \frac{P_t - P_s}{\frac{\rho V^2}{2}}$$



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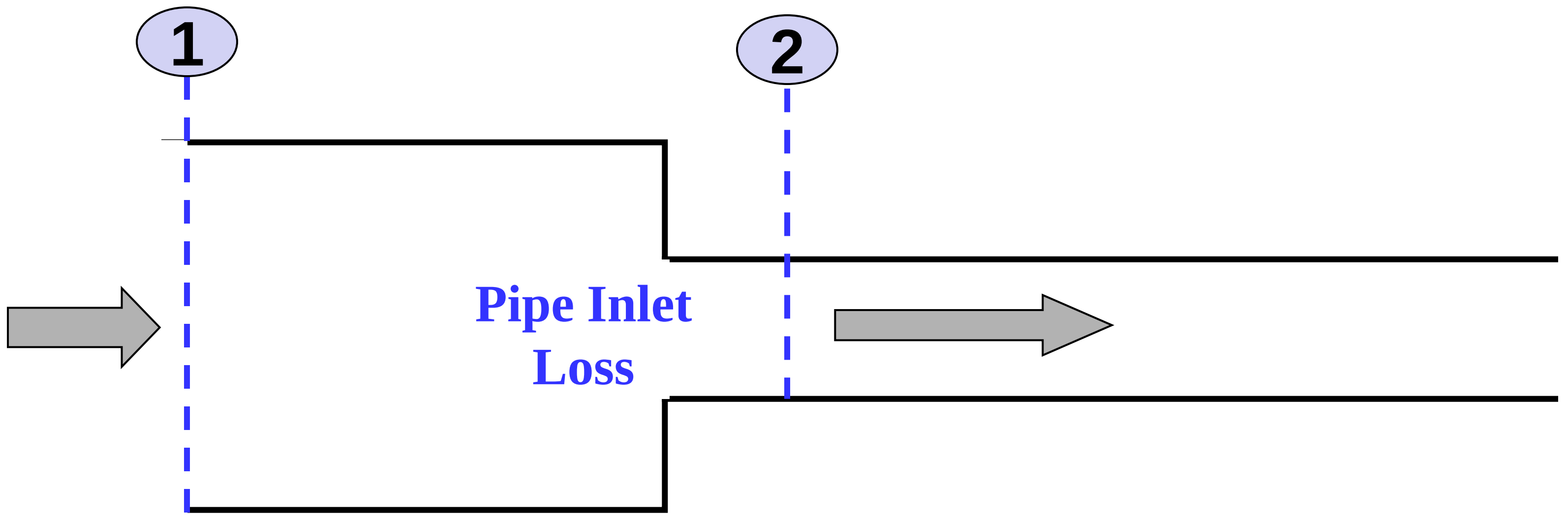
Loss Coefficient Versus Discharge Coefficient (1)

- Inlet Minor Loss (K_{inlet})



$$K_{inlet} = \frac{P_{t_1} - P_{t_2}}{P_{t_2} - P_{s_2}}$$

Loss Coefficient Versus Discharge Coefficient (2)

- Inlet Minor Loss (K_{inlet})

$$\boxed{K_{inlet} = \frac{P_{t_1} - P_{t_2}}{P_{t_2} - P_{s_2}}} \Rightarrow \boxed{P_{t_2} = \frac{P_{t_1} + K_{inlet} P_{s_2}}{(1 + K_{inlet})}}$$

- Incompressible Flow

For an incompressible flow

$$\boxed{P_{t_1} - P_{t_2} = \frac{K_{inlet}\, \rho V_2^2}{2}} \Rightarrow \boxed{\dot{m}_{K_{inlet}} = A_2 \rho V_2 = A_2 \sqrt{\frac{2\rho(P_{t_1} - P_{t_2})}{K_{inlet}}}}$$

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Loss Coefficient Versus Discharge Coefficient (3)

$$\dot{m}_{K_{inlet}} = A_2 \rho V_2 = A_2 \sqrt{\frac{2\rho}{K_{inlet}}\left[P_{t_1} - \frac{\left(P_{t_1} + K_{inlet} P_{s_2}\right)}{(1+K_{inlet})}\right]}$$

$$= A_2 \sqrt{\frac{2\rho}{K_{inlet}}\left[\frac{K_{inlet}\left(P_{t_1} - P_{s_2}\right)}{(1+K_{inlet})}\right]}$$

$$\boxed{\dot{m}_{K_{inlet}} = A_2 \sqrt{\frac{2\rho\left(P_{t_1} - P_{s_2}\right)}{(1+K_{inlet})}}}$$

Loss Coefficient Versus Discharge Coefficient (4)

- **Discharge Coefficient (C_d)**

$$C_d = \frac{\dot{m}_{actual}}{\dot{m}_{ideal}}$$

  - **Incompressible Flow**

    - **Bernoulli's Equation between Sections 1 and 2 yields**

$$P_{s_1} + \frac{\rho V_1^2}{2} = P_{s_2} + \frac{\rho V_2^2}{2} = P_{t_1}$$

which gives

$$V_2 = \sqrt{\frac{2(P_{t_1} - P_{s_2})}{\rho}}$$

Loss Coefficient Versus Discharge Coefficient (5)

❑ Discharge Coefficient (C_d)

❖ Incompressible Flow

$$\dot{m}_{C_d} = \rho A_2 C_d V_2 = \rho A_2 C_d \sqrt{\frac{2(P_{t_1} - P_{s_2})}{\rho}} = A_2 C_d \sqrt{2\rho(P_{t_1} - P_{s_2})}$$

$$\boxed{\dot{m}_{C_d} = A_2 C_d \sqrt{2\rho(P_{t_1} - P_{s_2})}}$$

where

$C_d \equiv$ Discharge Coefficient

Loss Coefficient Versus Discharge Coefficient (6)

- Relation Between Loss Coefficient (K_{inlet}) and Discharge Coefficient (C_d) for an Incompressible Flow

$$\dot{m}_{K_{inlet}} = A_2\sqrt{\frac{2\rho\left(P_{t_1} - P_{s_2}\right)}{\left(1+K_{inlet}\right)}}$$

$$\dot{m}_{C_d} = A_2 C_d \sqrt{2\rho(P_{t_1} - P_{s_2})}$$

Thus, we obtain the following relation between K_{inlet} and C_d

$$C_d = \frac{1}{\sqrt{1+K_{inlet}}}$$

Loss Coefficient Versus Discharge Coefficient (7)

Relation Between Loss Coefficient (K_{inlet}) and Discharge Coefficient (C_d) for a Compressible Flow

❖ Mass Flow Rate Calculation Using K_{inlet}

$$K_{inlet} = \frac{P_{t_1} - P_{t_2}}{P_{t_2} - P_{s_2}}$$

$$P_{t_2} = \frac{P_{t_1} + K_{inlet} P_{s_2}}{(1 + K_{inlet})}$$

$$\frac{P_{t_2}}{P_{s_2}} = \frac{\dfrac{P_{t_1}}{P_{s_2}} + K_{inlet}}{(1 + K_{inlet})}$$

$$\dot{m}_{K_{inlet}} = \frac{F_{f_{t_2}} A_2 P_{t_2}}{\sqrt{T_{t_2}}}$$

where

$$F_{f_{t_2}} = M_2 \sqrt{\frac{\kappa}{R\left(1 + \dfrac{(\kappa - 1)M_2^2}{2}\right)^{\frac{\kappa+1}{\kappa-1}}}} = M_2 \sqrt{\frac{\kappa}{R}\left(\frac{P_{s_2}}{P_{t_2}}\right)^{\frac{\kappa+1}{\kappa}}}$$

and

$$M_2 = \sqrt{\frac{2}{\kappa - 1}\left[\left(\frac{P_{t_2}}{P_{s_2}}\right)^{\frac{\kappa-1}{\kappa}} - 1\right]}$$

Loss Coefficient Versus Discharge Coefficient (8)

- Relation Between Loss Coefficient (K_{inlet}) and Discharge Coefficient (C_d) for a Compressible Flow

  - Mass Flow Rate Calculation Using C_d

$$\boxed{\dot{m}_{C_d} = \frac{\tilde{F}_{f_{t_2}} C_d A_2 P_{t_1}}{\sqrt{T_{t_1}}}}$$

where

$$\boxed{\tilde{F}_{f_{t_2}} = \tilde{M}_2 \sqrt{\frac{\kappa}{R\left(1 + \frac{(\kappa - 1)\tilde{M}_2^2}{2}\right)^{\frac{\kappa+1}{\kappa-1}}}} = \tilde{M}_2 \sqrt{\frac{\kappa}{R}\left(\frac{P_{s_2}}{P_{t_1}}\right)^{\frac{\kappa+1}{\kappa}}}}$$

and

$$\boxed{\tilde{M}_2 = \sqrt{\frac{2}{\kappa - 1}\left[\left(\frac{P_{t_1}}{P_{s_2}}\right)^{\frac{\kappa-1}{\kappa}} - 1\right]}}$$

Loss Coefficient Versus Discharge Coefficient (9)

- **Relation Between Loss Coefficient (K_{inlet}) and Discharge Coefficient (C_d) for a Compressible Flow**
  - Let's assume $T_{t_1} = T_{t_2}$

    Then for $\dot{m}_{K_{inlet}} = \dot{m}_{C_d}$, we obtain

$$\frac{F_{f_{t_2}} A_2 P_{t_2}}{\sqrt{T_{t_2}}} = \frac{\tilde{F}_{f_{t_2}} C_d A_2 P_{t_1}}{\sqrt{T_{t_1}}} \quad \text{or} \quad \tilde{F}_{f_{t_2}} C_d P_{t_1} = F_{f_{t_2}} P_{t_2}$$

$$\boxed{C_d = \frac{F_{f_{t_2}} P_{t_2}}{\tilde{F}_{f_{t_2}} P_{t_1}}} \quad \text{or} \quad \boxed{C_d = \left(\frac{F_{f_{t_2}}}{\tilde{F}_{f_{t_2}}}\right)\left(\frac{P_{t_1} + K_{inlet} P_{s_2}}{P_{t_1}(1 + K_{inlet})}\right)}$$

Loss Coefficient Versus Discharge Coefficient (10)

- **Relation Between Loss Coefficient (K_{inlet}) and Discharge Coefficient (C_d) for a Compressible Flow**

$$\boxed{C_d = \left(\frac{F_{f_{t_2}}}{\tilde{F}_{f_{t_2}}}\right)\left(\frac{P_{t_1} + K_{inlet} P_{s_2}}{P_{t_1}(1 + K_{inlet})}\right)}$$

$$\boxed{C_d = \left(\frac{F_{f_{t_2}}}{\tilde{F}_{f_{t_2}}}\right)\left(\frac{\dfrac{P_{t_1}}{P_{s_2}} + K_{inlet}}{\dfrac{P_{t_1}}{P_{s_2}}(1 + K_{inlet})}\right)}$$

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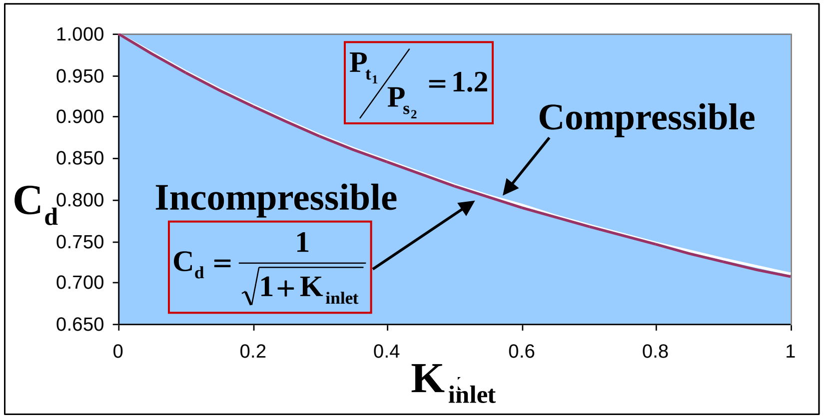
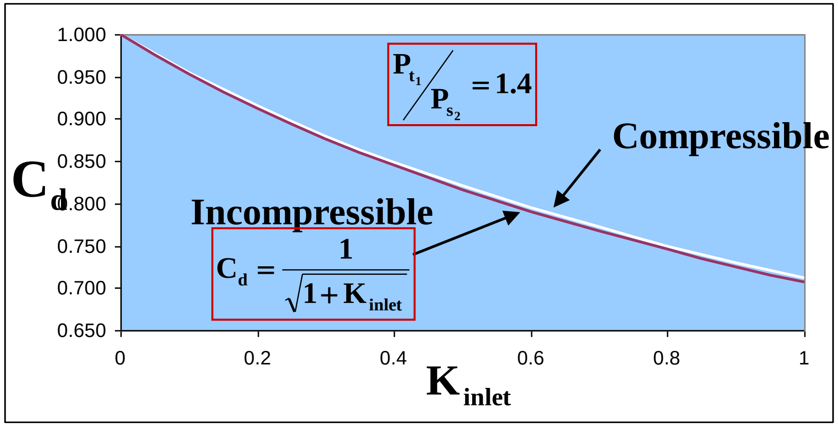
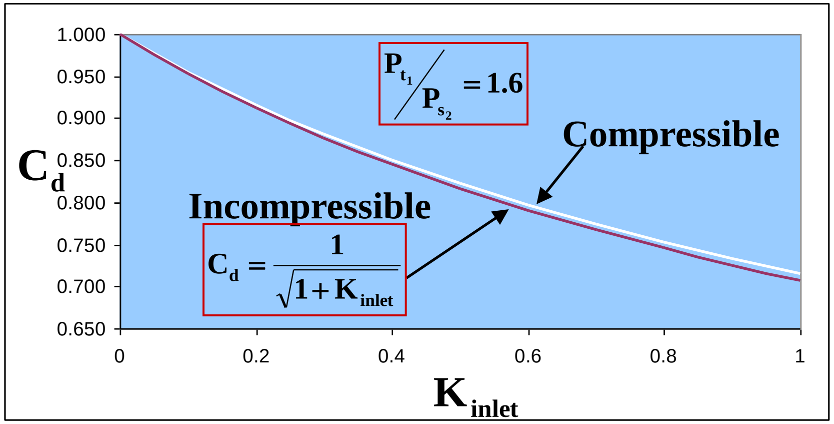
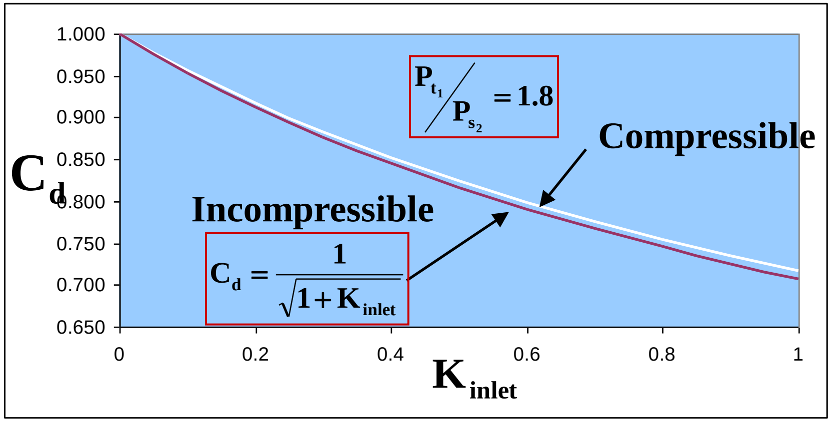
Loss Coefficient Versus Discharge Coefficient (11)

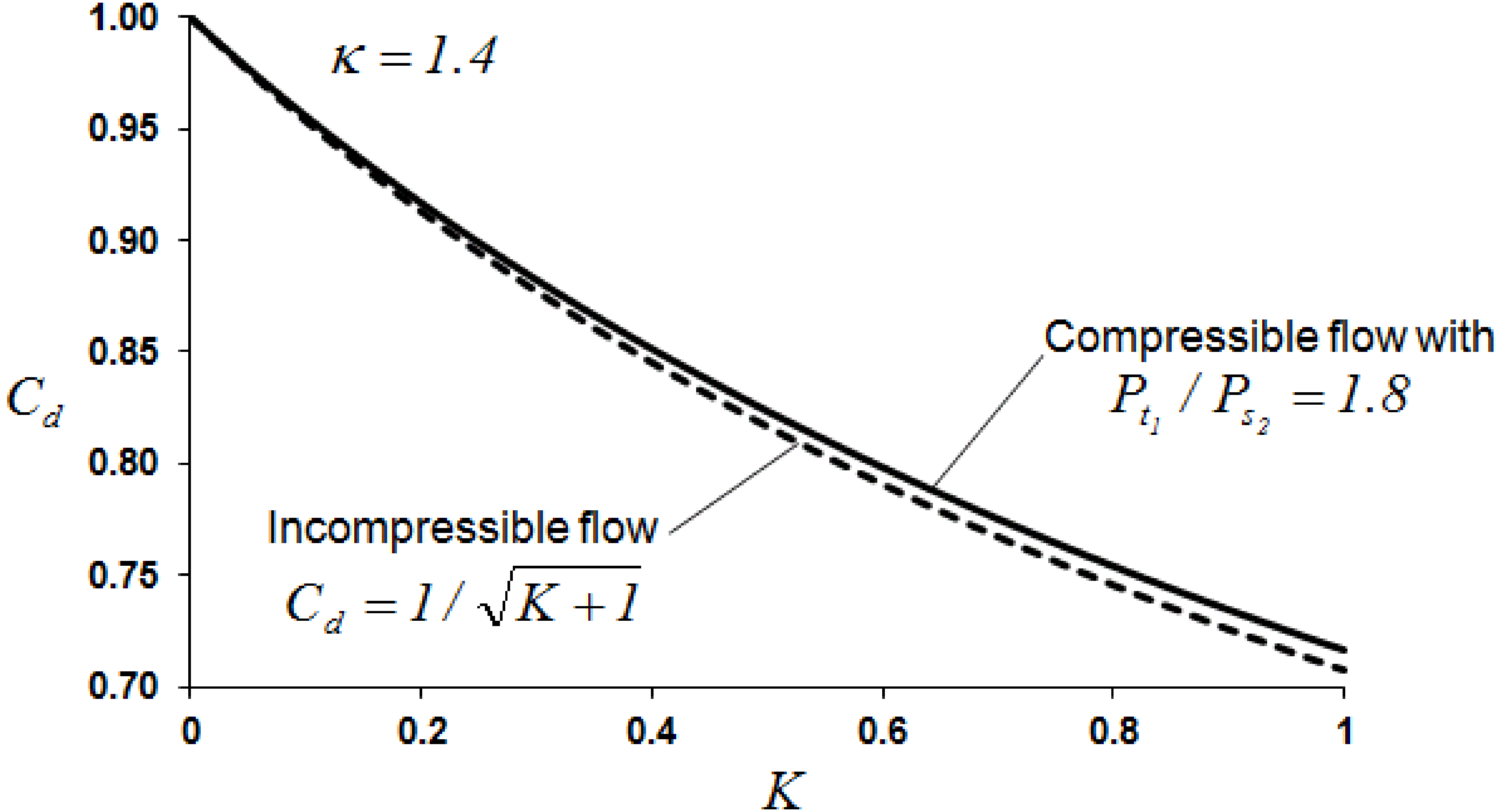








Loss Coefficient Versus Discharge Coefficient (12)



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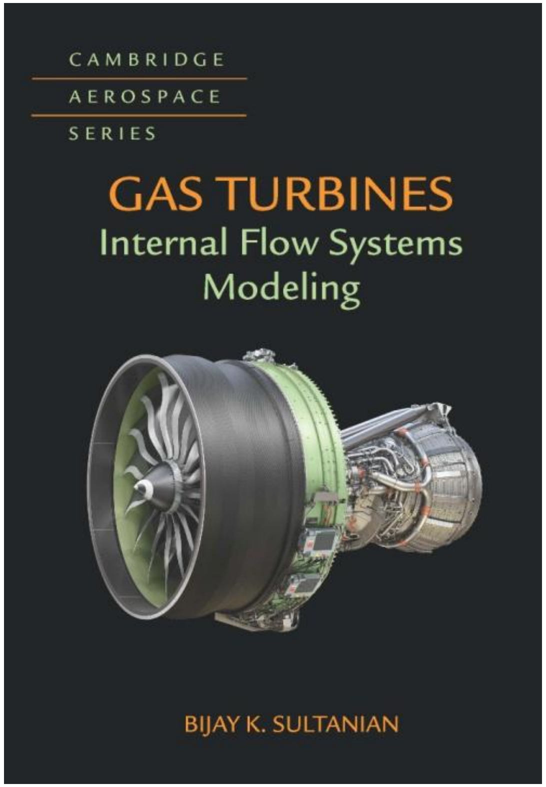
QUESTIONS?

THANK YOU!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Physics-Based Modeling of Gas Turbine Secondary Air Systems

Module 3: Special Concepts of Secondary Air Systems – Part II

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Phoenix, Arizona, USA
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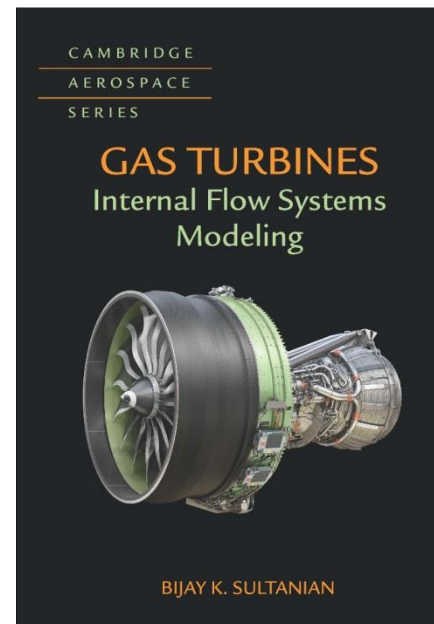


Module 3

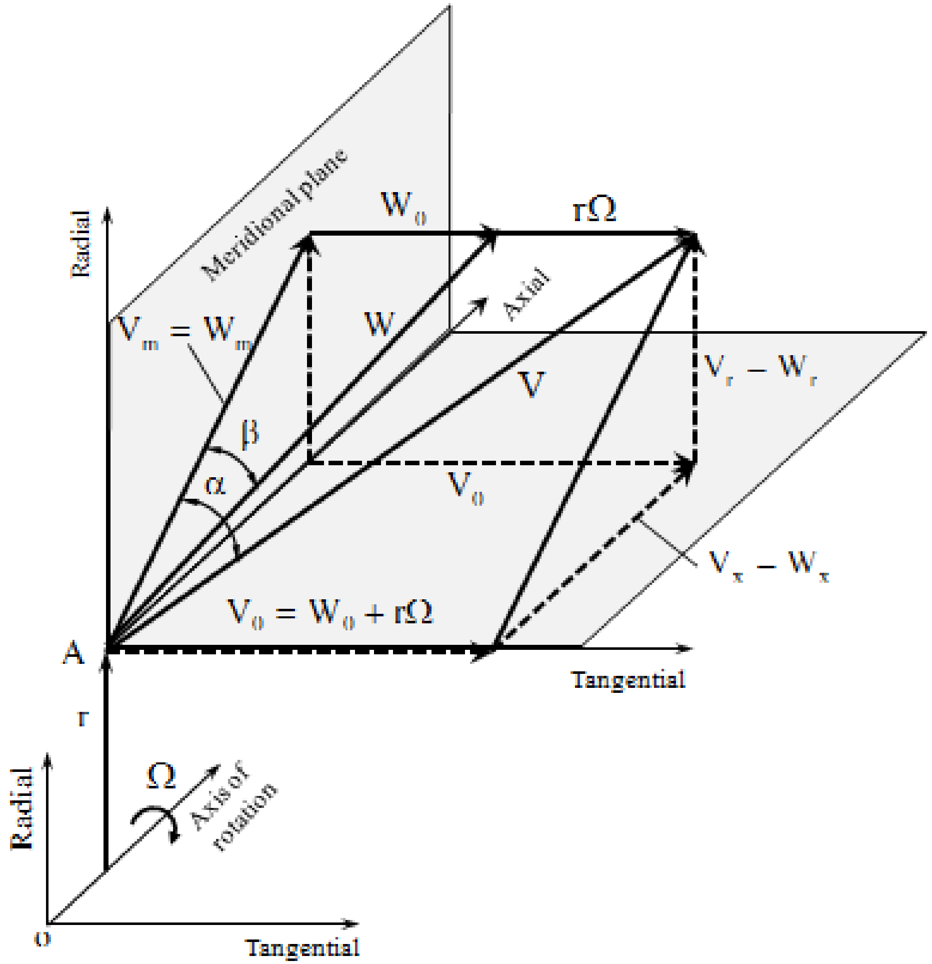
Special Concepts of Secondary Air Systems – Part II

Module 3

Special Concepts of Secondary Air Systems – Part II

- **Stator/Rotor reference frames**
- **Euler’s turbomachinery equation**
- **Rothalpy**
- **Preswirl system**
- **Rotor disk pumping**

Stator/Rotor Reference Frames



$$V_x = W_x$$

$$V_r = W_r$$

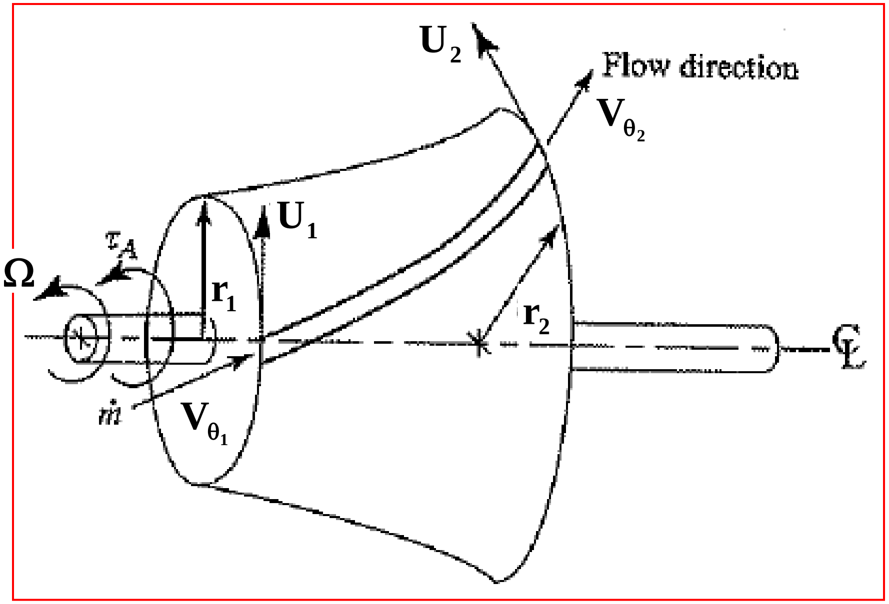
$$V_m = W_m$$

$$V_\theta = W_\theta + r\Omega$$

Euler's Turbomachinery Equation (1)

(Rotational Speed: Ω)

Control Volume for a General Turbomachine



- From Torque and Angular Momentum Balance:

$$\boxed{\tau_A = \dot{m}(r_2 V_{\theta_2} - r_1 V_{\theta_1})}$$

- Power = (Torque) x (Angular Speed)

$$\dot{W}_P = \dot{m}(r_2 V_{\theta_2} - r_1 V_{\theta_1})\Omega$$

$$U_1 = r_1\Omega \qquad U_2 = r_2\Omega$$

$$\boxed{\dot{W}_P = \dot{m}(U_2 V_{\theta_2} - U_1 V_{\theta_1})}$$

Euler's Turbomachinery Equation (2)

- Combining with SFEE, we obtain the Euler's Turbomachinery Equation (for unit mass flow rate of the working fluid)

$$\boxed{h_{t_2} - h_{t_1} = U_2 V_{\theta_2} - U_1 V_{\theta_1}}$$

- **For Turbines:**

$$\boxed{h_{t_2} < h_{t_1} \quad \text{or} \quad U_2 V_{\theta_2} < U_1 V_{\theta_1}}$$

- **For Compressors and Pumps:**

$$\boxed{h_{t_2} > h_{t_1} \quad \text{or} \quad U_2 V_{\theta_2} > U_1 V_{\theta_1}}$$

Rothalpy (1)

- Euler's Turbomachinery Equation

$$\boxed{h_{t_2} - h_{t_1} = U_2 V_{\theta_2} - U_1 V_{\theta_1}}$$

- Rearranging Euler's Turbomachinery Equation Yields

$$\boxed{h_{t_2} - U_2 V_{\theta_2} = h_{t_1} - U_1 V_{\theta_1} = I} \qquad \boxed{I \equiv \text{Rothalpy}}$$

- Rothalpy in Stator Reference Frame (SRF)

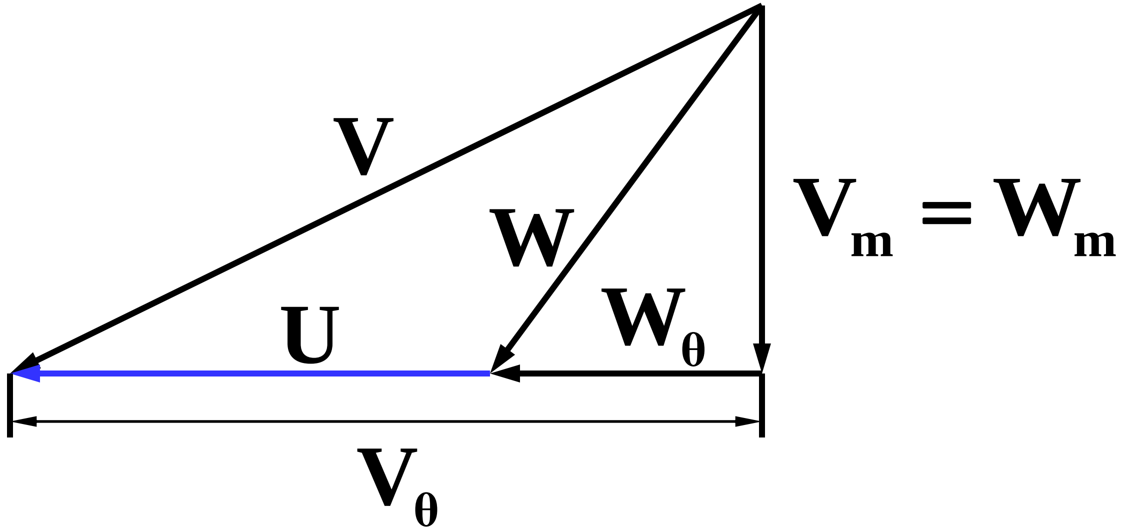
$$\boxed{I = h_s + \frac{V^2}{2} - U V_\theta}$$

Rothalpy (2)

❑ Rothalpy in Rotor Reference Frame (RRF)

$$V_x = W_x$$

$$V_r = W_r$$



$$\boxed{W_\theta = V_\theta - U} \qquad \boxed{V_\theta = W_\theta + U}$$

$$\boxed{V^2 = V_m^2 + V_\theta^2 = W_m^2 + W_\theta^2 + 2W_\theta U + U^2}$$

Rothalpy (3)

Rothalpy in Rotor Reference Frame (RRF)

$$I = h_s + \frac{V^2}{2} - UV_\theta$$

$$= h_s + \frac{W_x^2 + W_\theta^2 + 2W_\theta U + U^2}{2} - U(W_\theta + U)$$

$$= h_s + \frac{W_x^2 + W_\theta^2}{2} + W_\theta U + \frac{U^2}{2} - U\,W_\theta - U^2$$

Thus

$$\boxed{I = h_s + \frac{W^2}{2} - \frac{U^2}{2} = h_{t_R} - \frac{U^2}{2} = c_p T_{t_R} - \frac{U^2}{2}}$$

For a rotor with no heat transfer:

$$\boxed{I_1 = I_2} \quad \text{or} \quad \boxed{h_{t_{R1}} - \frac{U_1^2}{2} = h_{t_{R2}} - \frac{U_2^2}{2}} \quad \text{or} \quad \boxed{c_P T_{t_{R1}} - \frac{U_1^2}{2} = c_P T_{t_{R2}} - \frac{U_2^2}{2}}$$

Rothalpy (4)

❖ Equating Rothalpy Expressions in SRF and RRF

$$\boxed{h_{t_S} - UV_\theta = h_{t_R} - \frac{U^2}{2}}$$

$$\boxed{c_P T_{t_S} - UV_\theta = c_P T_{t_R} - \frac{U^2}{2}}$$

$$\boxed{h_{t_S} - r\Omega V_\theta = h_{t_R} - \frac{r^2\Omega^2}{2}}$$

$$\boxed{c_P T_{t_S} - r\Omega V_\theta = c_P T_{t_R} - \frac{r^2\Omega^2}{2}}$$

❖ Conversion of Total Temperature between SRF and RRF

$$\boxed{T_{t_R} = T_{t_S} + \frac{U(U - 2V_\theta)}{2c_P}}$$

$$\boxed{T_{t_S} = T_{t_R} + \frac{U(U + 2W_\theta)}{2c_P}}$$

Rothalpy (5)

❖ **Conversion of Total Pressure between SRF and RRF**

$$\frac{P_{t_R}}{P_s} = \left(\frac{T_{t_R}}{T_s}\right)^{\frac{\kappa}{\kappa-1}}$$

$$\frac{P_{t_S}}{P_s} = \left(\frac{T_{t_S}}{T_s}\right)^{\frac{\kappa}{\kappa-1}}$$

$$P_{t_R} = P_s \left\{\frac{T_{t_S}}{T_s} + \frac{U(U-2V_\theta)}{2c_P T_s}\right\}^{\frac{\kappa}{\kappa-1}}$$

$$P_{t_S} = P_s \left\{\frac{T_{t_R}}{T_s} + \frac{U(U+2W_\theta)}{2c_P T_s}\right\}^{\frac{\kappa}{\kappa-1}}$$

Rothalpy (6)

❑ Conversion of Total Pressure between SRF and RRF

Using the fact that both static pressure and static temperature are independent of the reference frame, we can write

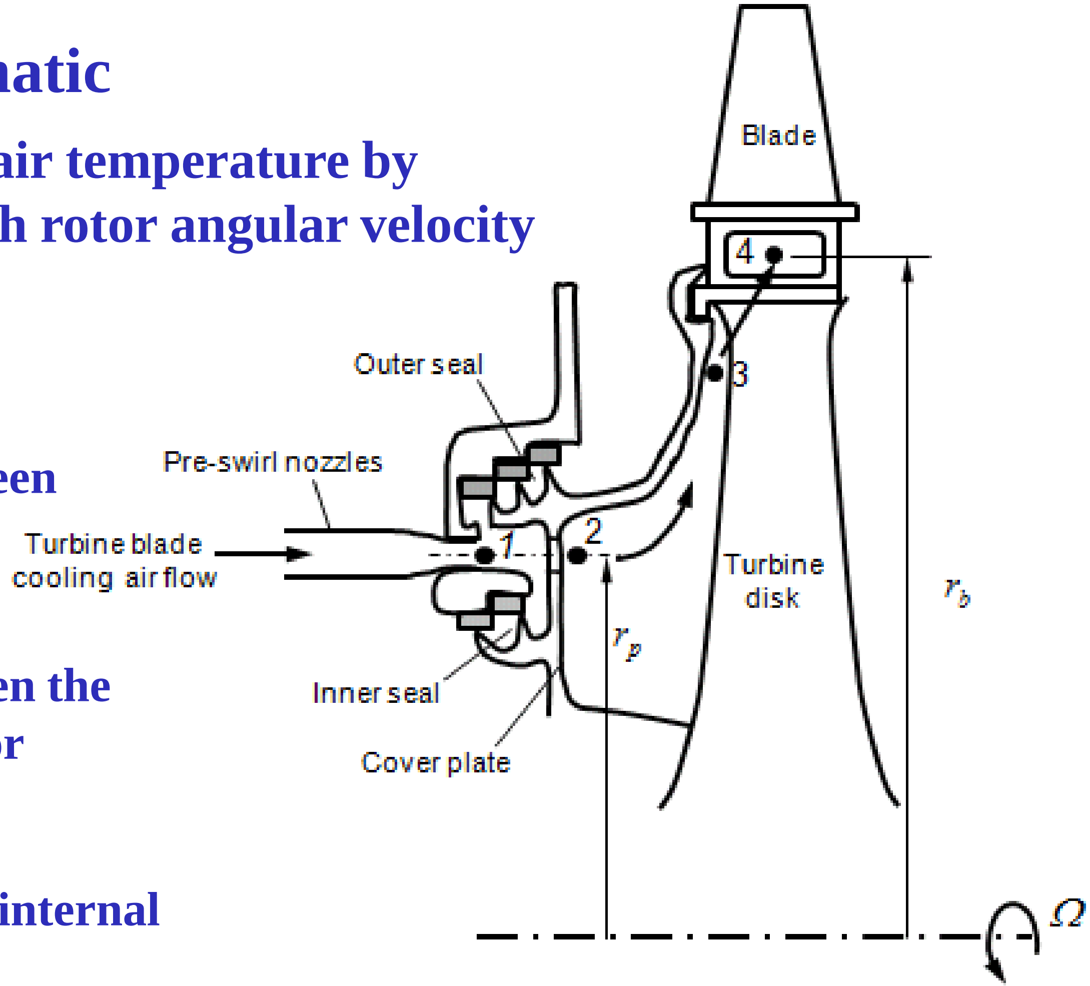
$$\boxed{\frac{P_{t_R}}{P_{t_S}} = \left(\frac{T_{t_R}}{T_{t_S}}\right)^{\frac{\kappa}{\kappa-1}}}$$

$$\boxed{P_{t_R} = P_{t_S}\left\{1+\frac{U(U-2V_\theta)}{2c_P T_{t_S}}\right\}^{\frac{\kappa}{\kappa-1}}} \qquad \boxed{P_{t_S} = P_{t_R}\left\{1+\frac{U(U+2W_\theta)}{2c_P T_{t_R}}\right\}^{\frac{\kappa}{\kappa-1}}}$$

Preswirl System (1)

❑ A Preswirl System Schematic

Role: To reduce blade cooling air temperature by preswirling it to match rotor angular velocity



Point 1: Preswirl nozzle exit

Point 2: Inlet to rotor cavity between the cover plate and the rotor disk

Point 3: Top of rotor cavity between the cover plate and the rotor disk

Point 4: Blade root (inlet to blade internal cooling)

Preswirl System (2)

Flow and Heat Transfer Modeling

Assuming adiabatic conditions with constant c_p and invoking the concept of rothalpy to relate total temperatures in SRF and RRF , we obtain at a point

$$\boxed{T_{t_S} - \frac{UV_\theta}{c_p} = T_{t_R} - \frac{U^2}{2c_p}}$$

$$\boxed{T_{t_R} = T_{t_S} - \frac{UV_\theta}{c_p} + \frac{U^2}{2c_p} = T_{t_S} + \frac{U^2}{2c_p}(1 - 2S_f)}$$

$$\boxed{S_f = \frac{V_\theta}{r_p \Omega}}$$

$$\boxed{T_{t_R} = T_{t_S} + \frac{r^2\Omega^2}{2c_p}(1 - 2S_f)}$$

$$\boxed{S_f = 0 : \; T_{t_R} = T_{t_S} + \frac{r^2\Omega^2}{2c_p}}$$

$$\boxed{S_f = 0.5 : \; T_{t_R} = T_{t_S}}$$

$$\boxed{S_f = 1 : \; T_{t_R} = T_{t_S} - \frac{r^2\Omega^2}{2c_p}}$$

Preswirl System (3)

❑ Flow and Heat Transfer Modeling

❖ Change in coolant air total temperature from point 1 (SRF) to point 4 (RRF)

$$\boxed{T_{t_{R_1}} = T_{t_{S_1}} + \frac{r_p^2\Omega^2}{2c_p}\left(1 - 2S_{f_1}\right)}$$

$$\boxed{T_{t_{R_4}} - \frac{r_b^2\Omega^2}{2c_p} = T_{t_{R_1}} - \frac{r_p^2\Omega^2}{2c_p}}$$

$$\boxed{\theta = \frac{T_{t_{S_1}} - T_{t_{R_4}}}{\left(\dfrac{r_b^2\Omega^2}{2c_p}\right)} = 2S_{f_1}\left(\frac{r_p}{r_b}\right)^2 - 1}$$

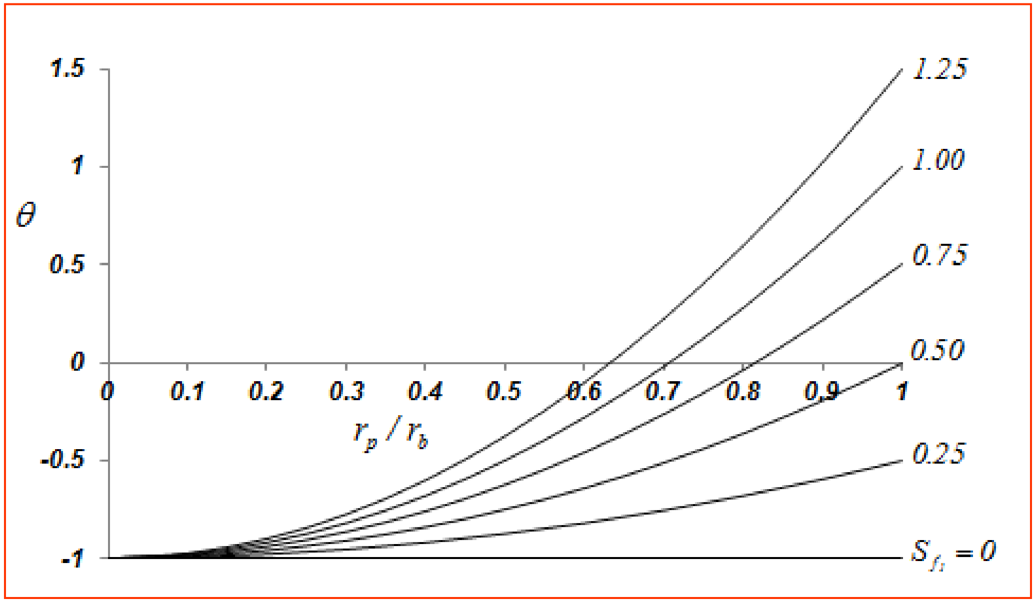
$\theta \equiv$ Blade cooling air reduction coefficient

$\dfrac{r_b^2\Omega^2}{2c_p} \equiv$ Dynamic temperature of solid-body rotation at r_b

Preswirl System (4)

Flow and Heat Transfer Modeling

- Change in coolant air total temperature from point 1 (SRF) to point 4 (RRF)
  - θ varies linearly with S_{f_1} and quadratically with r_p/r_b
  - Negative values of θ implies $T_{t_{R_4}} > T_{t_{S_1}}$
  - For $S_{f_1} = 1$, positive values of θ occur only for $r_p/r_b > 0.707$
  - Over-spinning ($S_{f_1} > 1$) the coolant air must be balanced against excess reduction in static pressure



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Preswirl System (5)

Flow and Heat Transfer Modeling

Turbine work loss coefficient

$$\boxed{T_{t_{S_1}} - \frac{r_p^2 \Omega^2 S_{f_1}}{c_p} = T_{t_{S_4}} - \frac{r_b^2 \Omega^2}{c_p}}$$

$$\boxed{\tilde{\theta} = \frac{T_{t_{S_4}} - T_{t_{S_1}}}{\left(\dfrac{r_b^2 \Omega^2}{2c_p}\right)} = 1 + \left\{1 - 2S_{f_1}\left(\frac{r_p}{r_b}\right)^2\right\} = 1 - \theta}$$

$\tilde{\theta} \equiv$ Turbine work loss coefficient

Preswirl System (6)

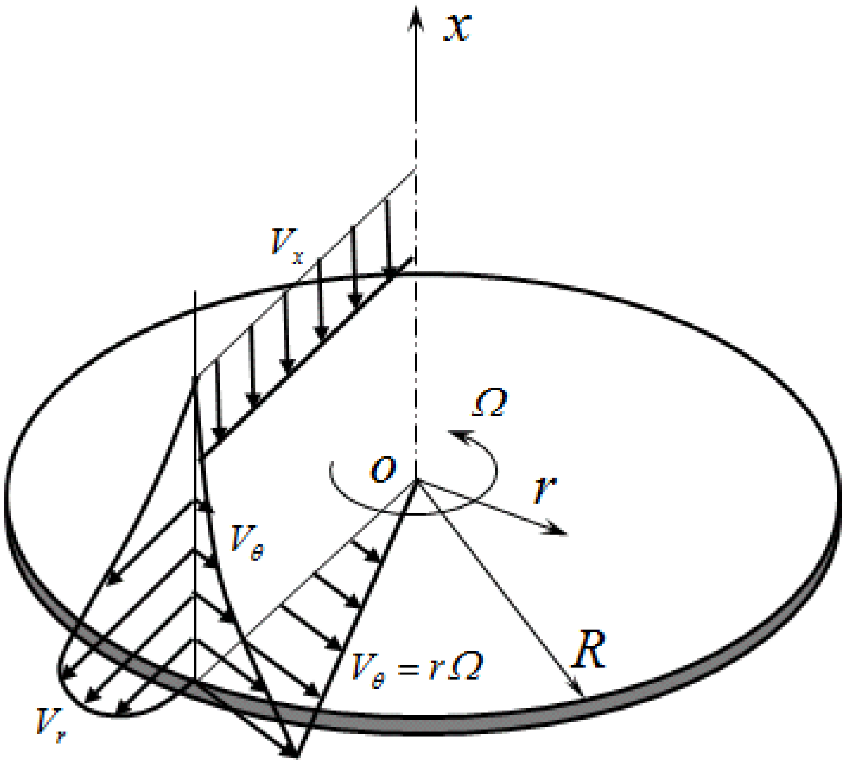
Flow and Heat Transfer Modeling

Concluding Remarks:

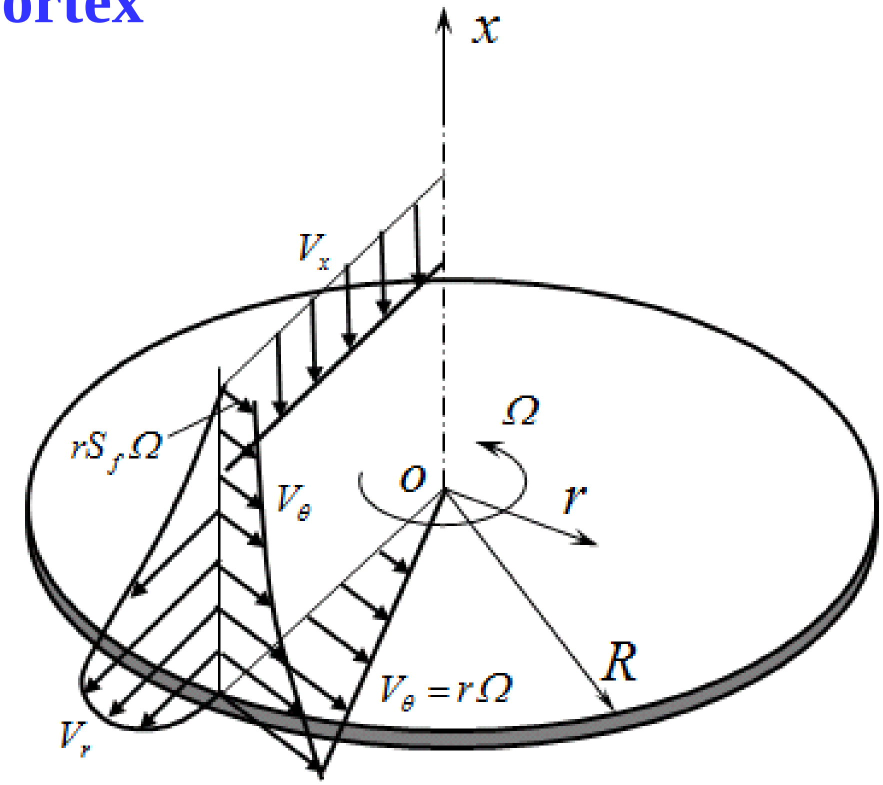
- The design goal of higher value of θ and lower value of $\tilde{\theta}$ is achieved simultaneously.
- For $\tilde{\theta} > 1$ or $\theta < 1$, the transfer system behaves like a compressor, and for $\tilde{\theta} < 1$ or $\theta > 1$, it behaves like a turbine.
- Must ensure adequate backflow margin to prevent any ingestion of hot gases through the blade film cooling holes.
- For the same value of S_{f_1}, V_{θ_1} will be higher, the higher the preswirl nozzle radius, requiring higher reduction in static pressure.
- Preswirl radial location determines rotor disk pressure distribution and impacts axial rotor thrust.

Rotor Disk Pumping (1)

- ❑ **Free Disk Pumping**



- ❑ **Disk Pumping beneath a Forced Vortex**



Rotor Disk Pumping (2)

Free Disk Pumping

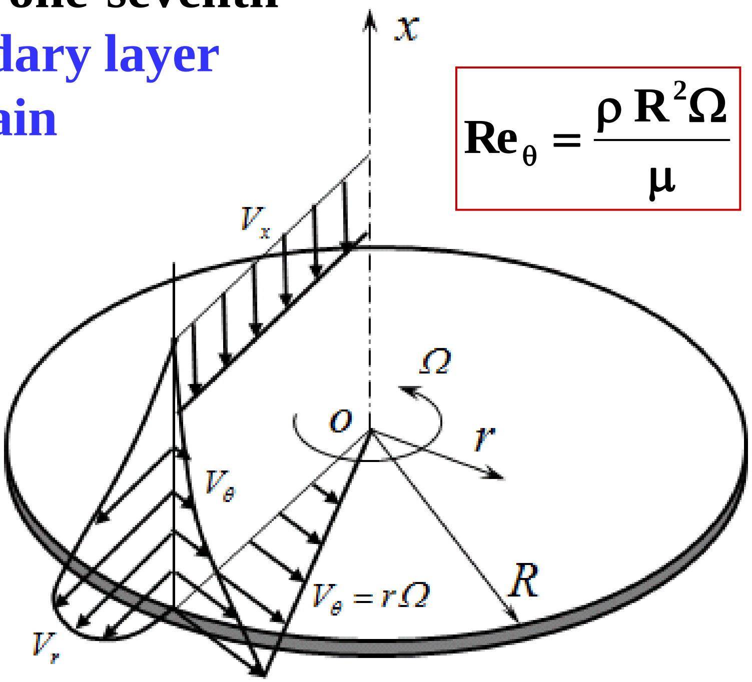
- Using von Karman's integral momentum equations for tangential and radial velocities with **one-seventh power-law** velocity profiles in the boundary layer and **logarithmic law of the wall**, we obtain

$$\dot{m}_{\text{free disk}}(R) = 0.219\,\rho\,R^3\,\Omega\,\mathrm{Re}_\theta^{-0.2}$$

$$\dot{m}_{\text{free disk}}(R) = 0.219\,\mu\,R\,\mathrm{Re}_\theta^{0.8}$$

$$\dot{m}_{\text{free disk}}(r) = 0.219\,\mu\,R\,\mathrm{Re}_\theta^{0.8}\left(\frac{r}{R}\right)^{2.6}$$

$$\mathrm{Re}_\theta = \frac{\rho\,R^2\Omega}{\mu}$$



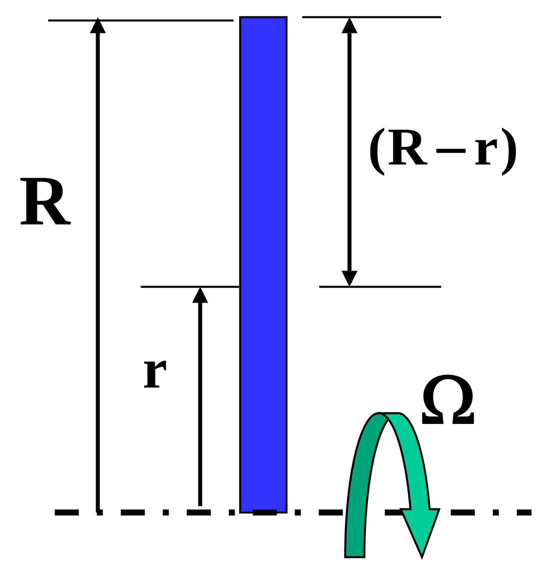
Rotor Disk Pumping (3)

Free Disk Pumping

$$\mathrm{Re}_\theta = \frac{\rho R^2 \Omega}{\mu}$$

$$\dot{m}_1 = \dot{m}_{\text{free disk}}(r) = 0.219\,\mu\, r\left(\frac{\rho r^2 \Omega}{\mu}\right)^{0.8}$$

$$\dot{m}_2 = \dot{m}_{\text{free disk}}(r) = 0.219\,\mu\, R\left(\frac{\rho R^2 \Omega}{\mu}\right)^{0.8}\left(\frac{r}{R}\right)^{2.6}$$



Note: $\dot{m}_1 = \dot{m}_2$

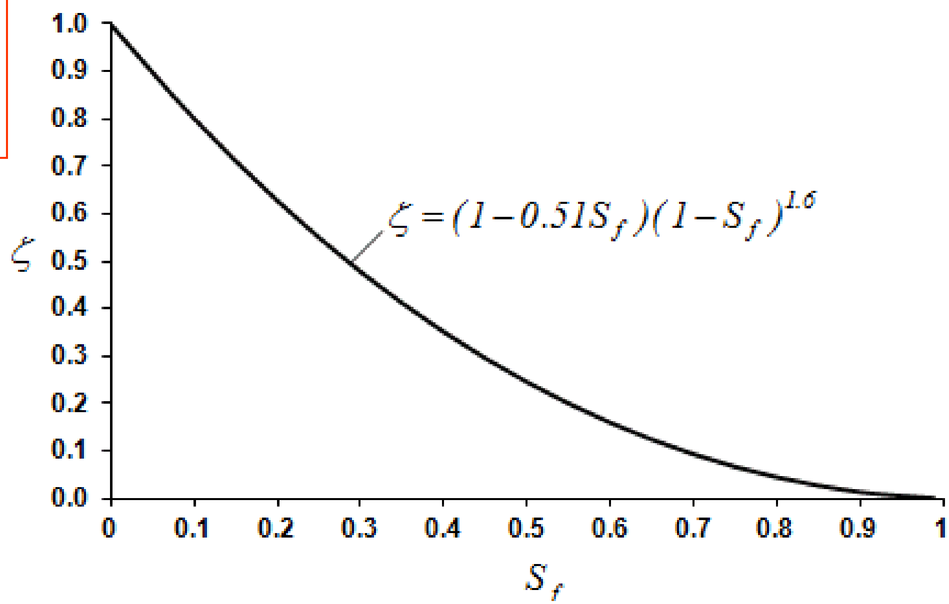
Rotor Disk Pumping (4)

Disk Pumping beneath a Forced Vortex

$$\boxed{\dot{m}_{\text{disk pump}}(r) = 0.219\,\mu\,R\,Re_{\theta}^{0.8}\left(\frac{r}{R}\right)^{2.6}\zeta}$$

$$\boxed{\zeta = (1 - 0.51\,S_f)(1 - S_f)^{1.6}}$$

$S_f \equiv$ Swirl Factor



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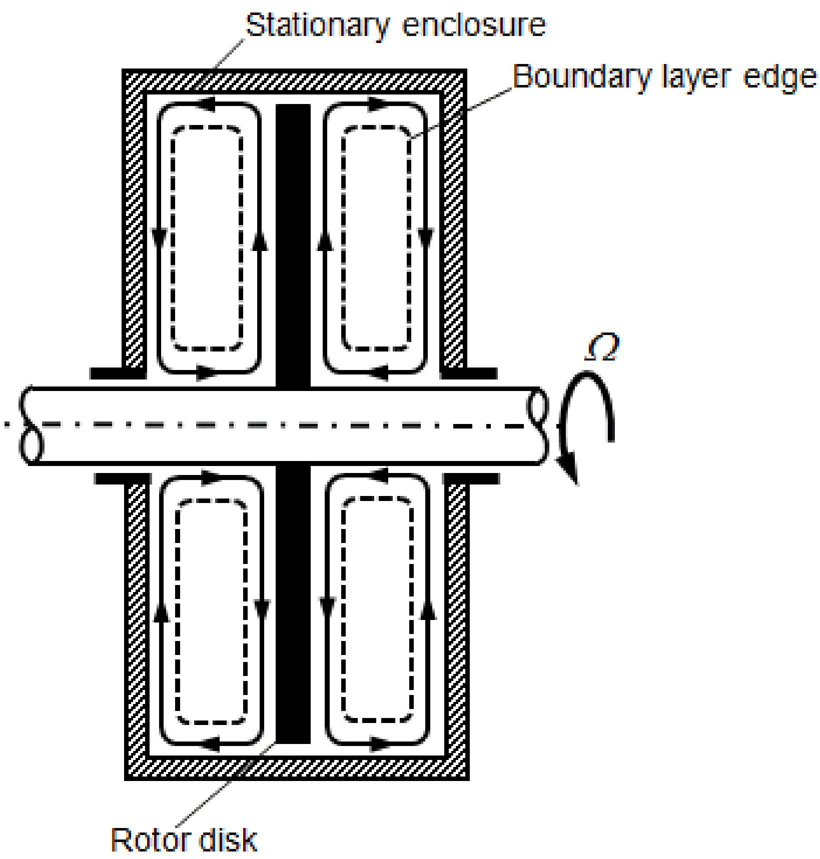
Rotor Disk Pumping (5)

❑ Rotor Disk in a Enclosed Cavity



QUESTIONS?

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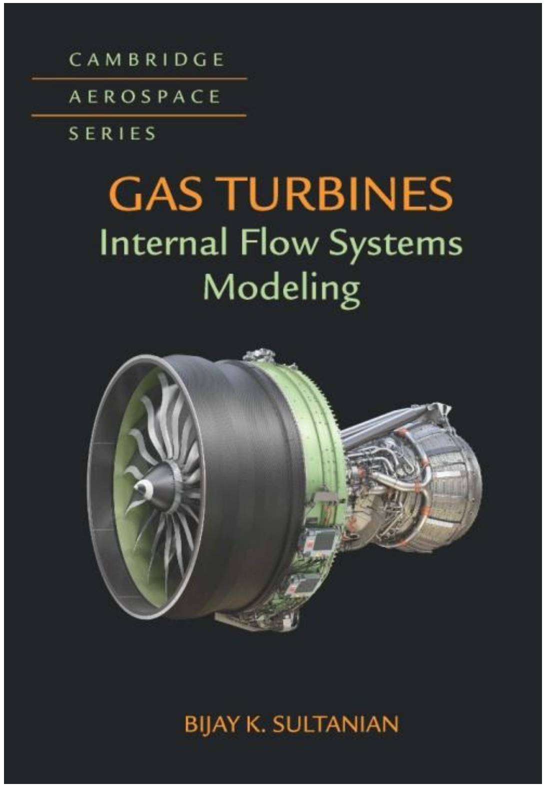
THANK YOU!



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Physics-Based Modeling of Gas Turbine Secondary Air Systems

Module 4: Physics-Based Modeling – Part I

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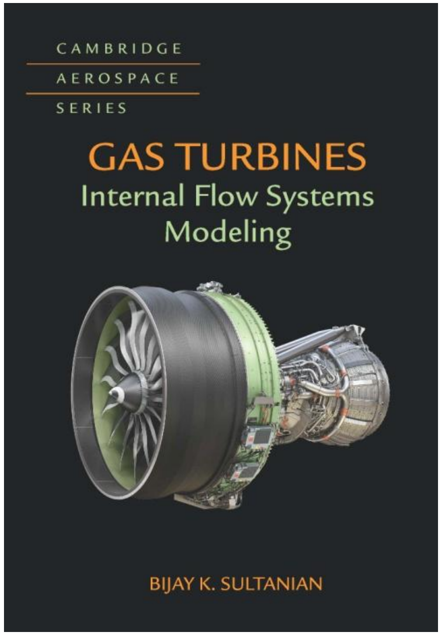
ASME 2019 TURBO EXPO

Phoenix, Arizona, USA
Sunday, June 16, 2019



Module 4

Physics-Based Modeling – Part I

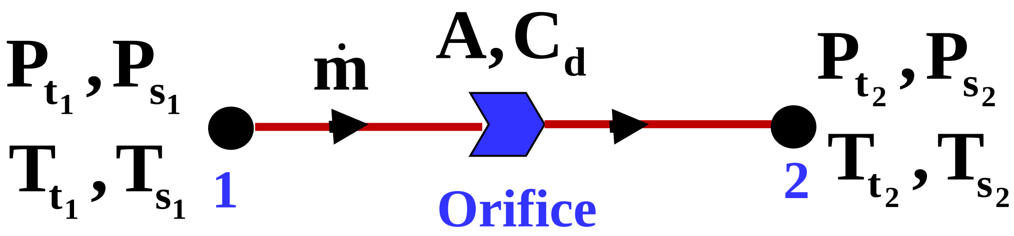
Module 4

Physics-Based Modeling – Part I

- Stationary and rotating orifices
- Stationary and rotating ducts
- Rotor-stator and rotor-rotor cavities
- Windage and swirl modeling in a general cavity
- Centrifugally-driven buoyant convection in compressor rotor cavity with and without bore flow

Element Modeling (1)

- Orifice



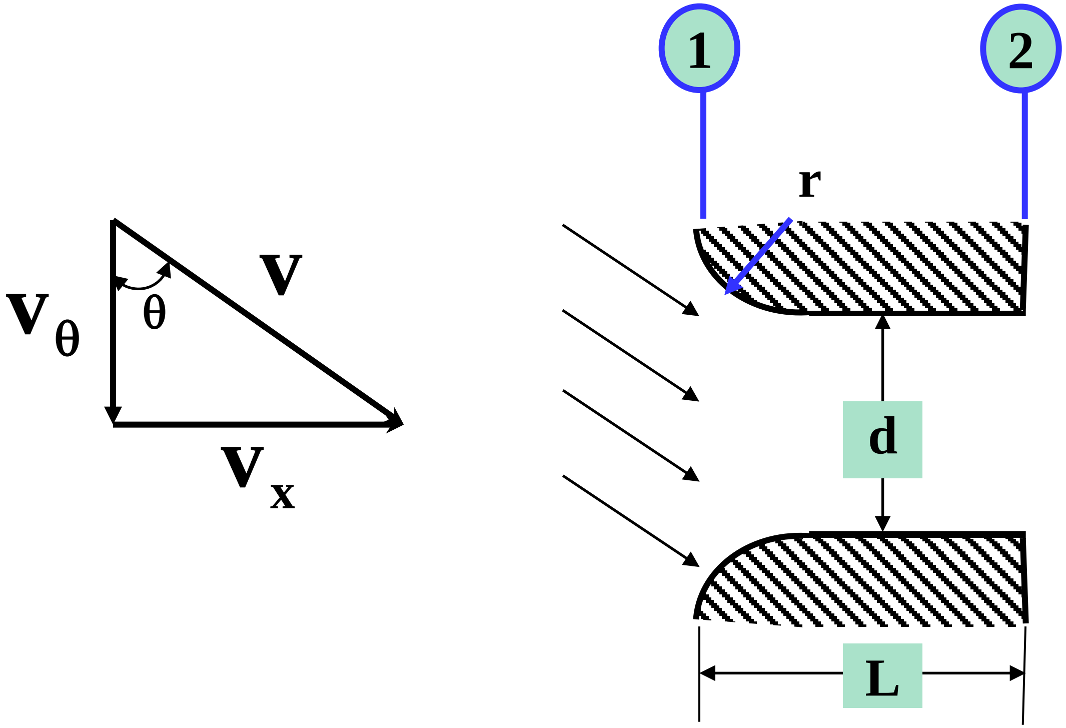
$$\boxed{\dot{m} = \frac{\hat{F}_{f_t} A C_d P_{t_1}}{\sqrt{R T_{t_1}}}}$$

where total-pressure flow function is given by

$$\boxed{\hat{F}_{f_t} = M \sqrt{\frac{\kappa}{\left(1 + \frac{(\kappa - 1) M^2}{2}\right)^{\frac{\kappa+1}{\kappa-1}}}}}$$

Note: Mach number M is a function of pressure ratio P_{t_1} / P_{s_1}

Stationary and Rotating Orifices (1)



$r \equiv$ Inlet corner radius

$d \equiv$ Orifice diameter

$L \equiv$ Orifice length

$Re \equiv$ Reynolds number

$$\boxed{Re = \frac{Vd}{\nu}}$$

Stationary and Rotating Orifices (2)

- Discharge Coefficient (C_d)

$$\dot{m}_{actual} = C_d\, \dot{m}_{ideal}$$

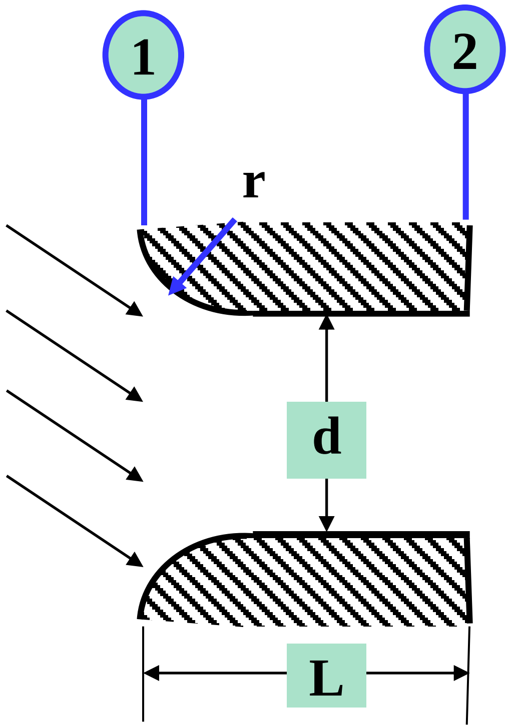
$$\dot{m}_{ideal} = A_2\left[\frac{1}{1-(A_2/A_1)^2}\right]^{1/2}\left[2\rho_1(P_{s1}-P_{s2})\right]^{1/2} Y$$

- Expansion factor for a sharp-edged orifice

$$Y = 1.0 - \left[0.41 + 0.35\left(\frac{d}{D}\right)^4\right]\frac{(P_{s1}-P_{s2})}{\kappa P_{s1}}$$

- Expansion factor for a nozzle

$$Y = \left[r^{2/\kappa}\left(\frac{\kappa}{\kappa-1}\right)\left(\frac{1-r^{(\kappa-1)/\kappa}}{1-r}\right)\frac{1-(A_1/A_2)^2}{1-(A_1/A_2)^2 r^{2/\kappa}}\right]^{1/2}$$



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Stationary and Rotating Orifices (3)

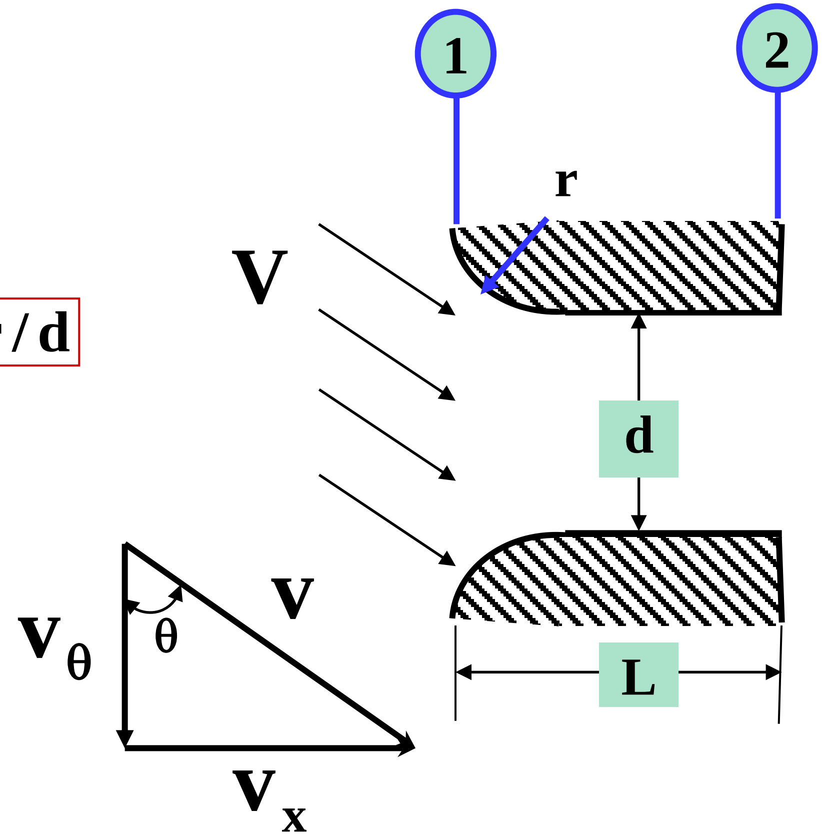
- Discharge Coefficient (C_d)

- Reynolds number $\boxed{Re = \dfrac{Vd}{\nu}}$

- Inlet corner radius-to-diameter ratio $\boxed{r/d}$

- Orifice length-to-diameter ratio $\boxed{L/d}$

- Relative tangential velocity (inlet flow angle of attack) $\boxed{V_\theta / V}$



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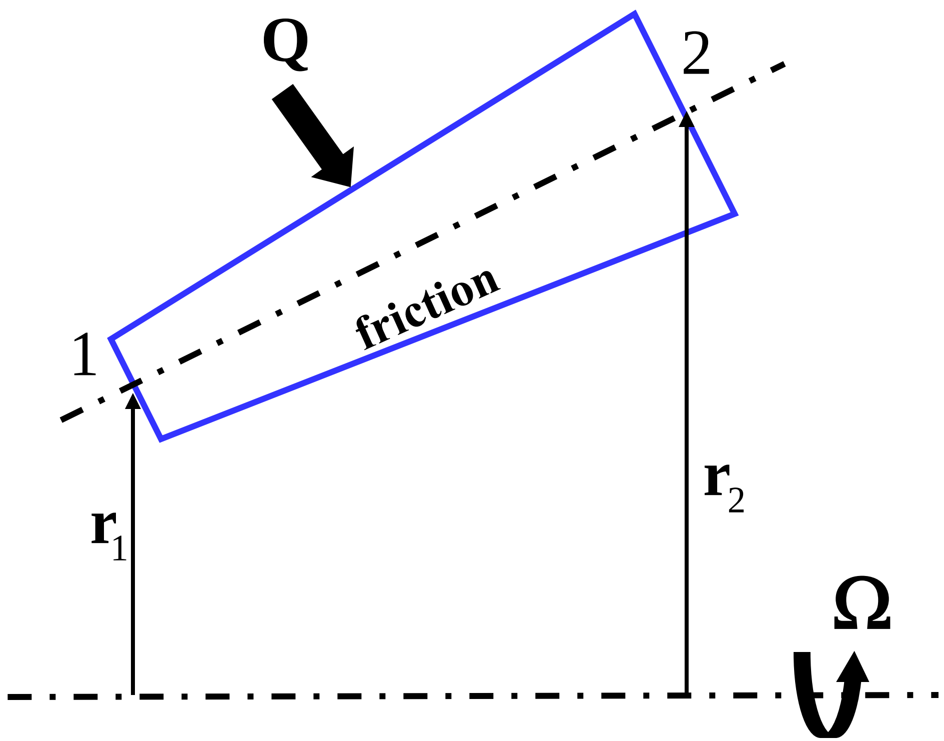
Stationary and Rotating Ducts (1)

- **Duct with Area Change, Friction, Heat Transfer, and Rotation**



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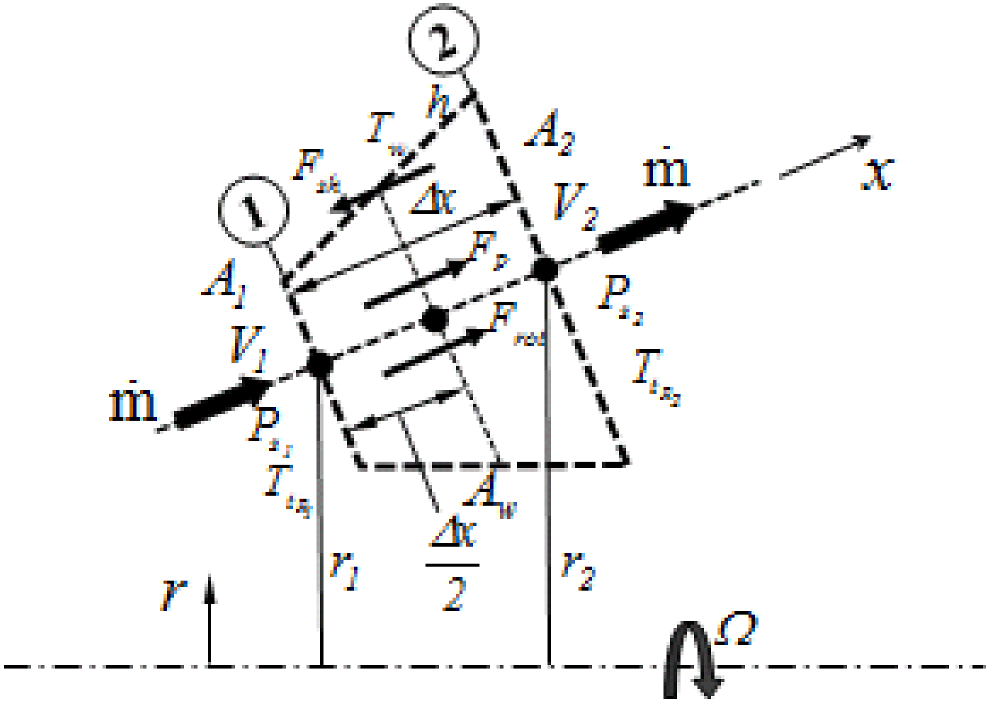
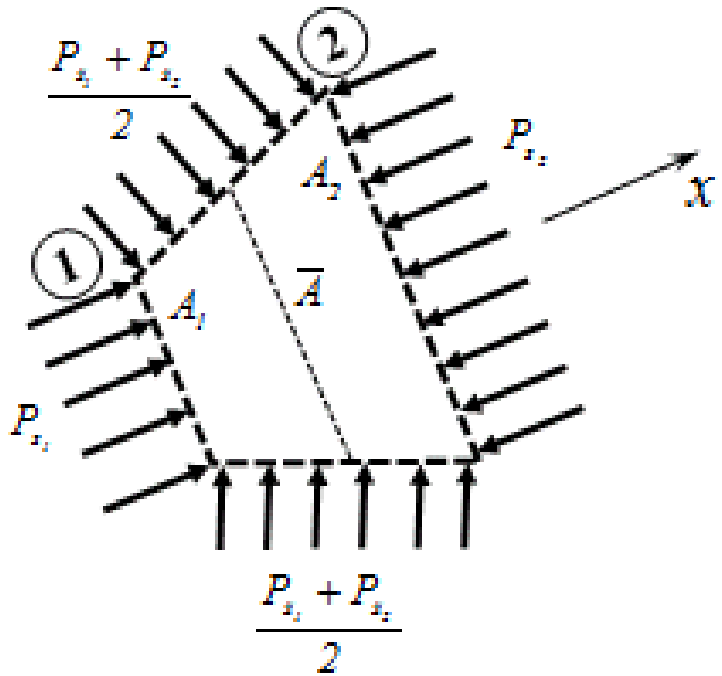
Stationary and Rotating Ducts (2)

❑ Duct with Area Change, Friction, Heat Transfer, and Rotation





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Stationary and Rotating Ducts (3)

Total-Pressure-Based Modeling Equations (Not Physics-Based!)

$$\left(P_{t_2} - P_{t_1}\right)_{\text{total}} = \left(P_{t_2} - P_{t_1}\right)_{\text{area change}} + \left(P_{t_2} - P_{t_1}\right)_{\text{rotation}} + \left(P_{t_2} - P_{t_1}\right)_{\text{heat transfer}} + \left(P_{t_2} - P_{t_1}\right)_{\text{friction}}$$

Note: $\left(P_{t_2} - P_{t_1}\right)_{\text{area change}} = 0$

- This approach ignores the nonlinear coupling between the momentum and energy equations inherent in a compressible flow.
- We cannot compute constant-area channel flow with both Friction and heat transfer by simply adding values from Fanno and Rayleigh flow tables!

Stationary and Rotating Ducts (4)

Static-Pressure-Based Modeling Equations

Continuity equation

$$\boxed{\dot{\mathbf{m}} = \rho_1 \mathbf{V}_1 \mathbf{A}_1 = \rho_2 \mathbf{V}_2 \mathbf{A}_2}$$

Total change in static pressure

$$\boxed{\mathbf{P}_{\mathbf{s}_2} - \mathbf{P}_{\mathbf{s}_1} = 0.5 * \bar{\rho}(\mathbf{V}_1^2 - \mathbf{V}_2^2) + \frac{\bar{\rho}\boldsymbol{\Omega}^2(\mathbf{r}_2^2 - \mathbf{r}_1^2)}{2} - \frac{\mathbf{f}\,\bar{\rho}\,\bar{\mathbf{V}}^2(\mathbf{x}_2 - \mathbf{x}_1)}{2\bar{\mathbf{D}}_{\mathbf{h}}}}$$

Stationary and Rotating Ducts (5)

Static-Pressure-Based Modeling Equations

- **Outlet total temperature**

$$\boxed{T_{t_{R_2}} = T_{t_{R_1}} e^{-\eta} + \left(\bar{T}_W - \frac{\Omega^2 (r_2 - r_1)^2}{c_P \eta^2} \right)(1 - e^{-\eta}) + \frac{\Omega^2 (r_2 - r_1)}{c_P \eta}(r_2 - r_1 e^{-\eta})}$$

$$\boxed{\eta = \frac{\bar{h} A_W}{\dot{m} c_P}} \qquad \boxed{\bar{h} \neq 0}$$

$$\boxed{T_{t_{R_2}} = T_{t_{R_1}} + \left(\Delta T_{t_R}\right)_{HT} + \left(\Delta T_{t_R}\right)_{rot} + \left(\Delta T_{t_R}\right)_{CCT}}$$

Stationary and Rotating Ducts (6)

Static-Pressure-Based Modeling Equations

Outlet total temperature

where

$$\boxed{\left(\Delta T_{t_R}\right)_{HT} = (T_{t_{R_2}} - T_{t_{R_1}})_{HT} = (T_w - T_{t_{R_1}})(1 - e^{-\eta})}$$

$$\boxed{\left(\Delta T_{t_R}\right)_{rot} = (T_{t_{R_2}} - T_{t_{R_1}})_{rot} = \frac{\Omega^2 (r_2^2 - r_1^2)}{2c_P}}$$

$$\boxed{\left(\Delta T_{t_R}\right)_{CCT} = -\frac{\Omega^2 (r_2 - r_1) r_1 \eta}{2c_P}}$$

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Stationary and Rotating Ducts (7)

Static-Pressure-Based Modeling Equations

- Outlet static temperature

$$T_{s_2} = T_{t_{R2}} - \frac{V_2^2}{2c_P}$$

- Outlet total pressure

$$P_{t_{R2}} = P_{s_2}\left(\frac{T_{t_{R2}}}{T_{s_2}}\right)^{\kappa/(\kappa-1)}$$

- Density at inlet and outlet

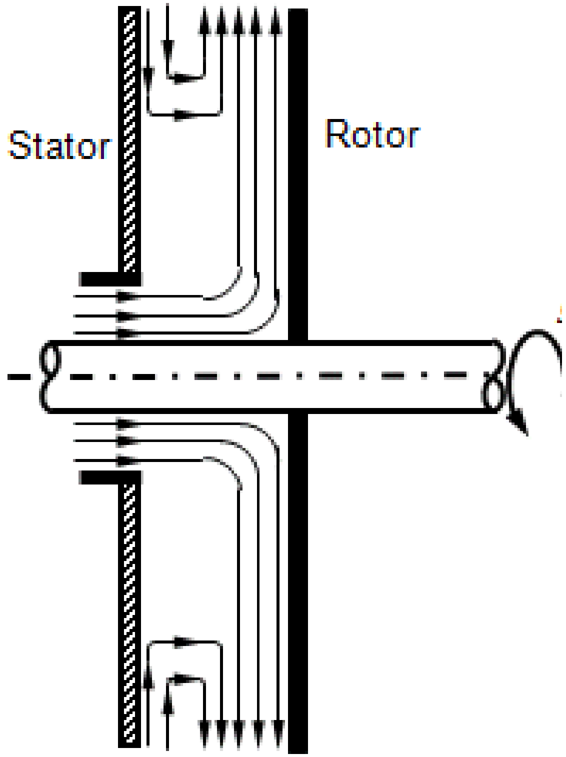
$$\rho_1 = \frac{P_{s_1}}{RT_{s_1}}$$

and

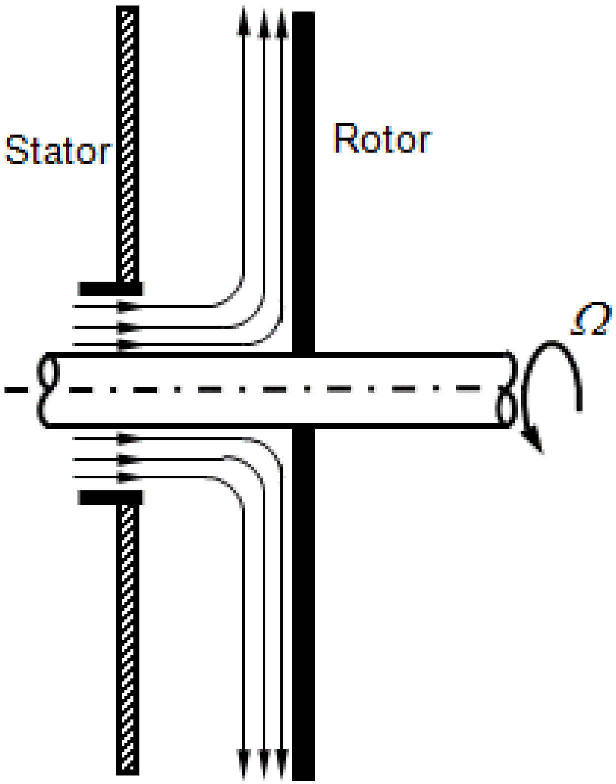
$$\rho_2 = \frac{P_{s_2}}{RT_{s_2}}$$

Rotor-Rotor and Rotor-Stator Cavities (1)

❑ Rotor-Stator Cavity with Radial Outflow



❖ Small outflow rate



❖ Large outflow rate

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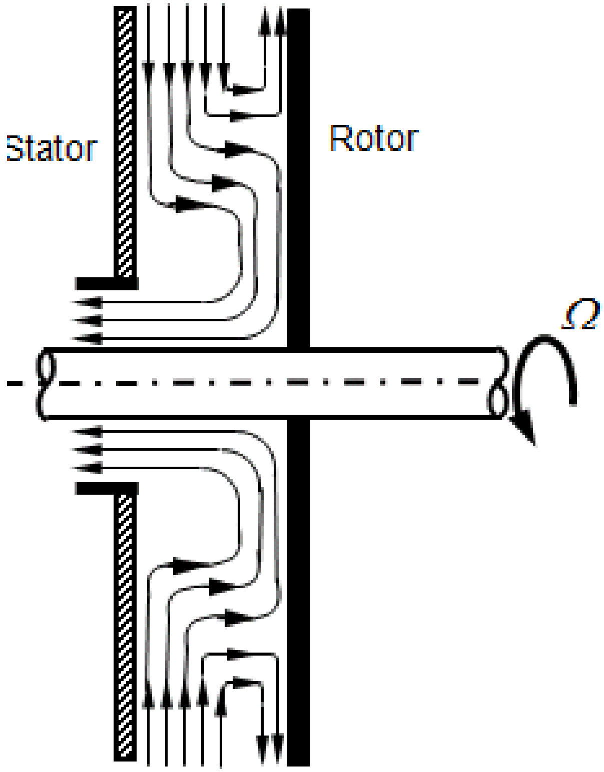
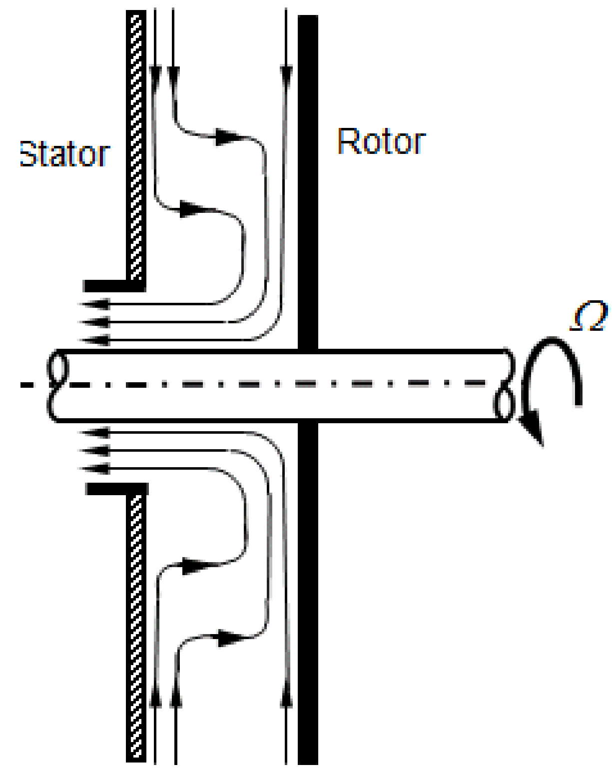
Rotor-Rotor and Rotor-Stator Cavities (2)

❑ Rotor-Stator Cavity with Radial Inflow



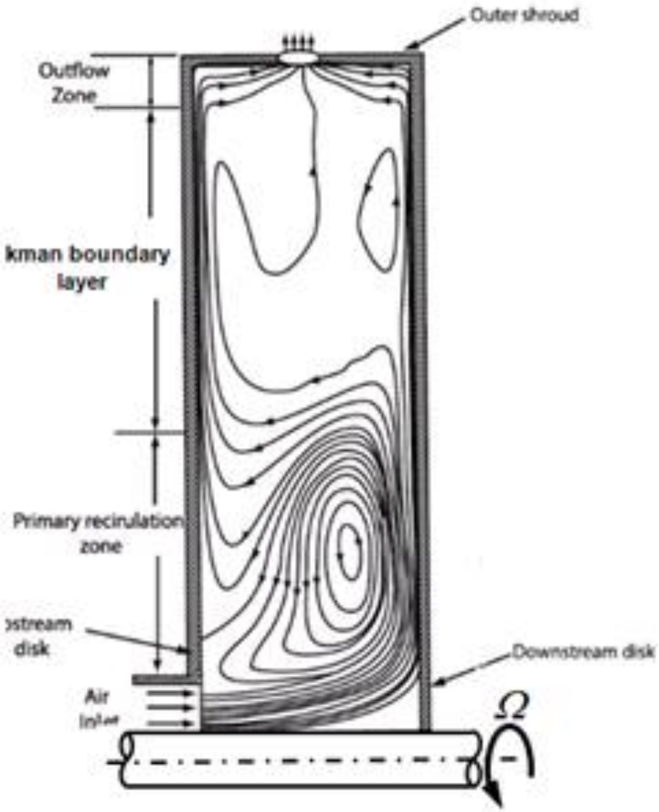
❖ Small inflow rate

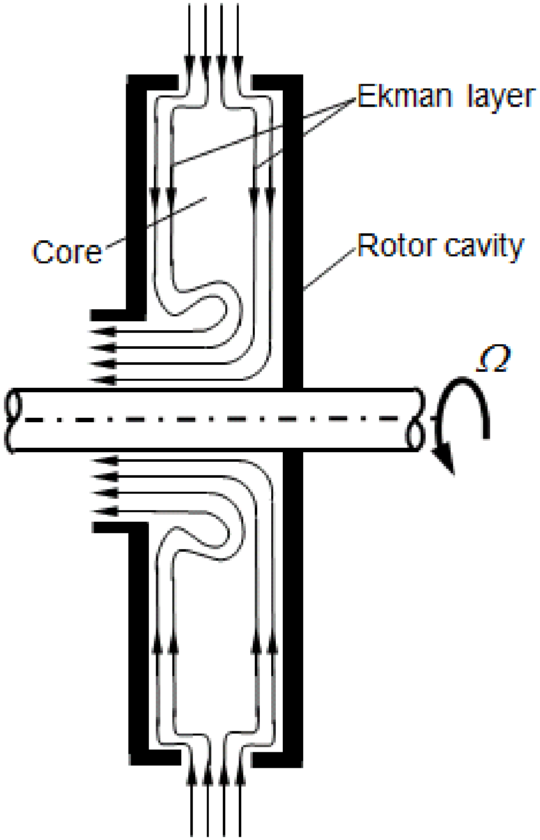


❖ Large inflow rate

Rotor-Rotor and Rotor-Stator Cavities (3)

❑ Rotating Cavity



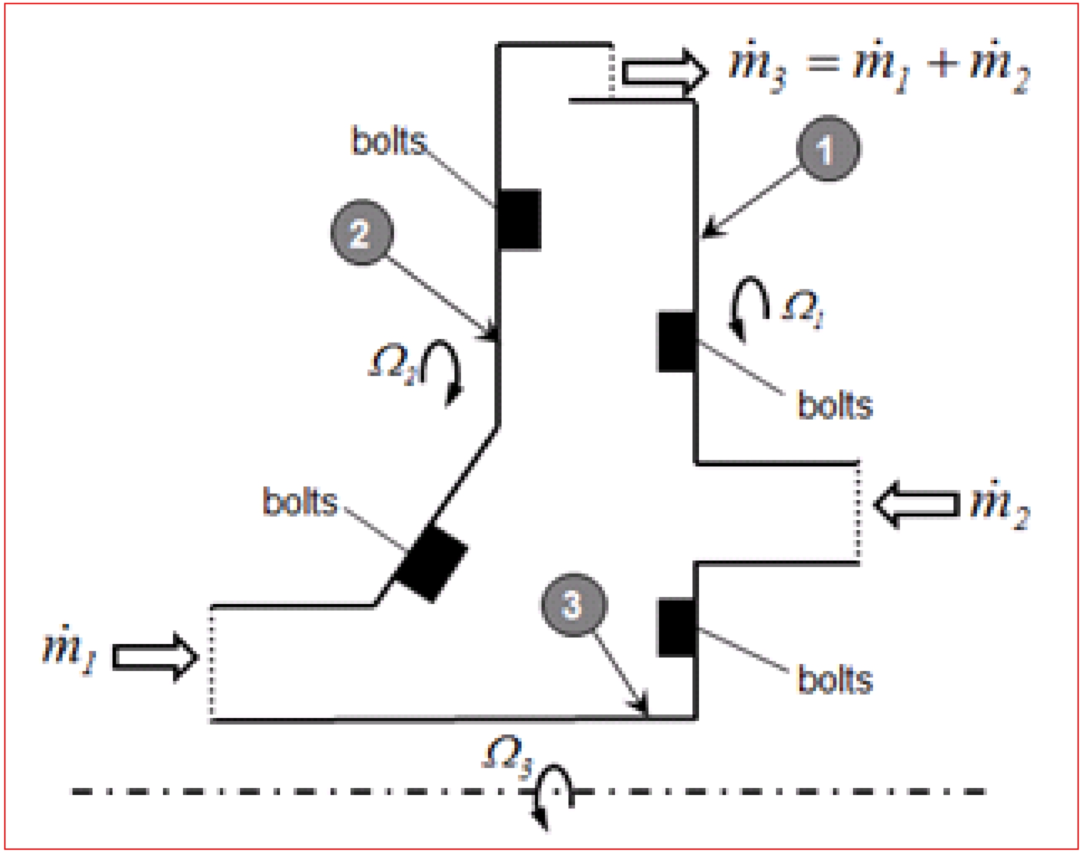


- ❖ Radial outflow
- ❖ Radial inflow

Windage and Swirl Modeling in a General Cavity (1)

Schematic of a General Gas Turbine Cavity and Its Key Features

- Multiple surfaces, which are rotating, stationary or counter-rotating.
- Each surface may be locally vertical, horizontal, and inclined.
- Multiple inflows and outflows with different swirl, temperature and pressure conditions
- Three-dimensional drag components, e.g., bolts



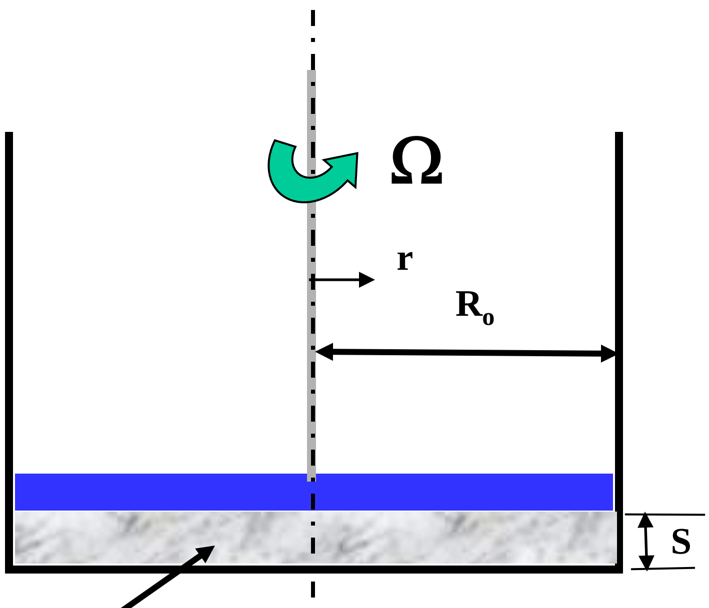
Windage and Swirl Modeling in a General Cavity (2)

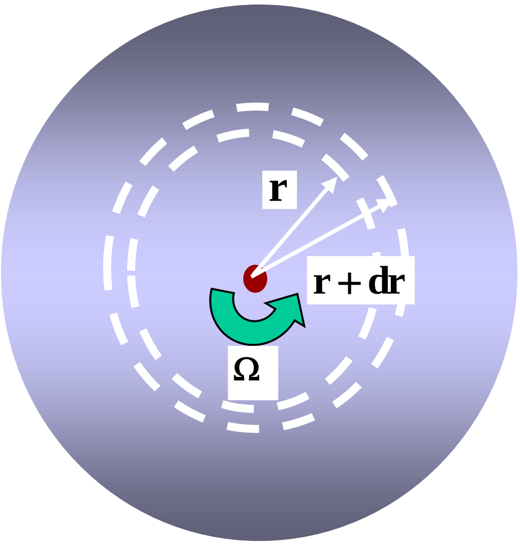
- Daily and Nece (1960)

"Chamber dimension effects on induced flow and friction resistance of enclosed rotating disks," J. Basic Engineering, Vol. 82, PP. 217-232.

$$\Gamma_{disk} = \int_0^{R_o} r\,\tau\,(2\pi\, r\, dr)$$

$\tau \equiv$ Tangential shear stress





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Windage and Swirl Modeling in a General Cavity (3)

❑ Daily and Nece (1960)

$$\tau = \frac{1}{2}\rho C_f (r\Omega)^2$$

$$\Gamma_{disk} = \frac{1}{5}\pi\rho C_f \Omega^2 R_o^5$$

$$= \frac{1}{2}\rho C_m \Omega^2 R_o^5$$

Thus,

$$\boxed{C_m = \frac{2\pi}{5} C_f = \frac{2\Gamma_{disk}}{\rho\Omega^2 R_o^5}}$$

Flow Regimes

I. Laminar, merged boundary layers

II. Laminar, separate boundary layers

III. Turbulent, merged boundary layers

IV. Turbulent, separate boundary layers

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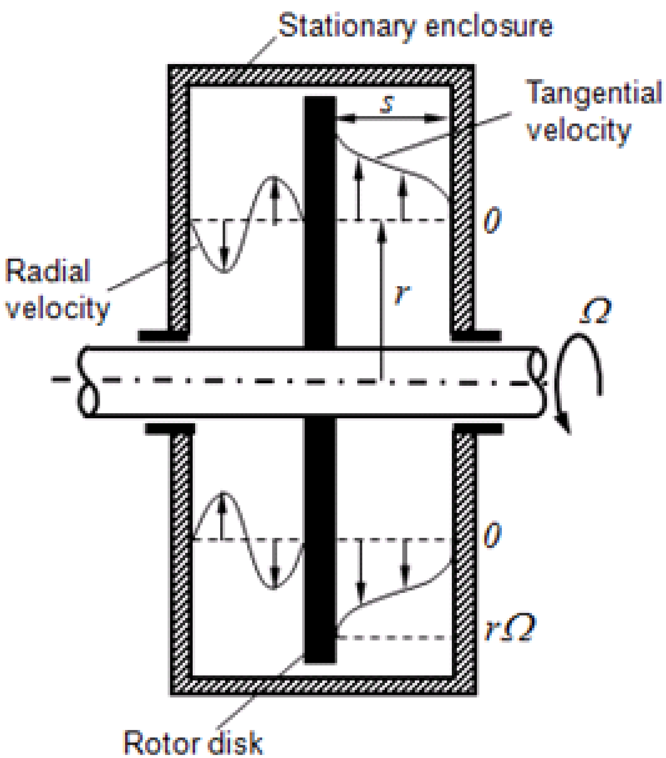
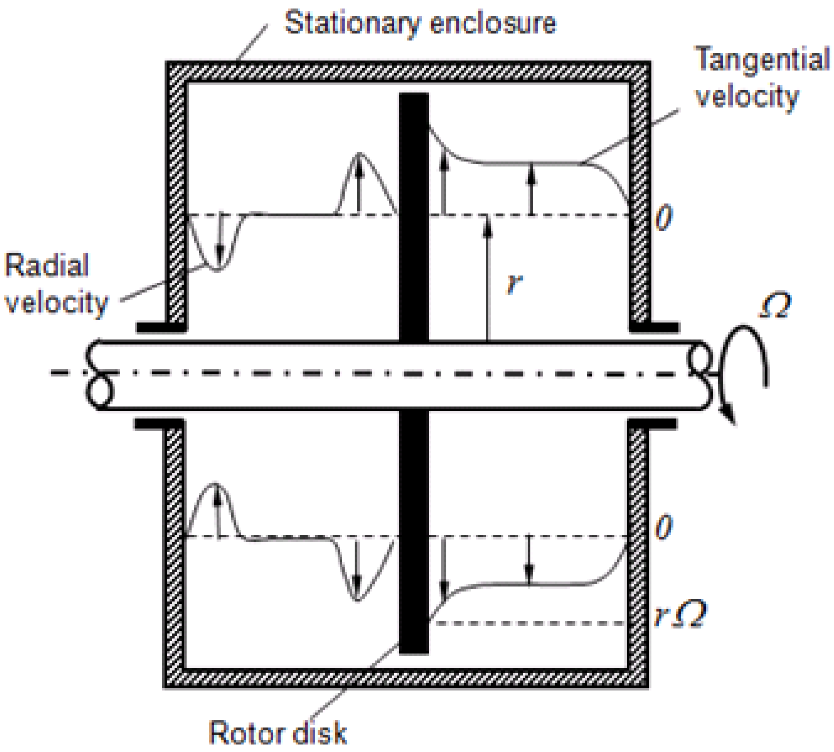
Windage and Swirl Modeling in a General Cavity (4)

❑ Daily and Nece (1960)



❖ Merged boundary layers (Regimes I and III)



❖ Separate boundary layers (Regimes II and IV)

Windage and Swirl Modeling in a General Cavity (5)

Daily and Nece (1960)

The four flow regimes are characterized by rotational Reynolds number Re and dimensionless gap parameter G.

$$\boxed{\mathrm{Re} = \Omega R_o^2 / \nu} \qquad \boxed{G = S / R_o}$$

I. Laminar, merged boundary layers

$$\boxed{C_m = \pi G^{-1}\,\mathrm{Re}^{-1}}$$

$$\begin{cases} G < 1.62\,\mathrm{Re}^{-5/11} \\ G < 188\,\mathrm{Re}^{-9/10} \end{cases}$$

II. Laminar, separate boundary layers

$$\boxed{C_m = 1.85 G^{1/10}\,\mathrm{Re}^{-1/2}}$$

$$\begin{cases} G > 1.62\,\mathrm{Re}^{-5/11} \\ G < 0.57\;10^{-6}\,\mathrm{Re}^{15/16} \\ \mathrm{Re} < 1.58\;10^{5} \end{cases}$$

III. Turbulent, merged boundary layers

$$\boxed{C_m = 0.040\, G^{-1/6}\,\mathrm{Re}^{-1/4}}$$

$$\begin{cases} G < 0.57\;10^{-6}\,\mathrm{Re}^{15/16} \\ G < 0.402\,\mathrm{Re}^{-3/16} \\ G > 188\,\mathrm{Re}^{-9/10} \end{cases}$$

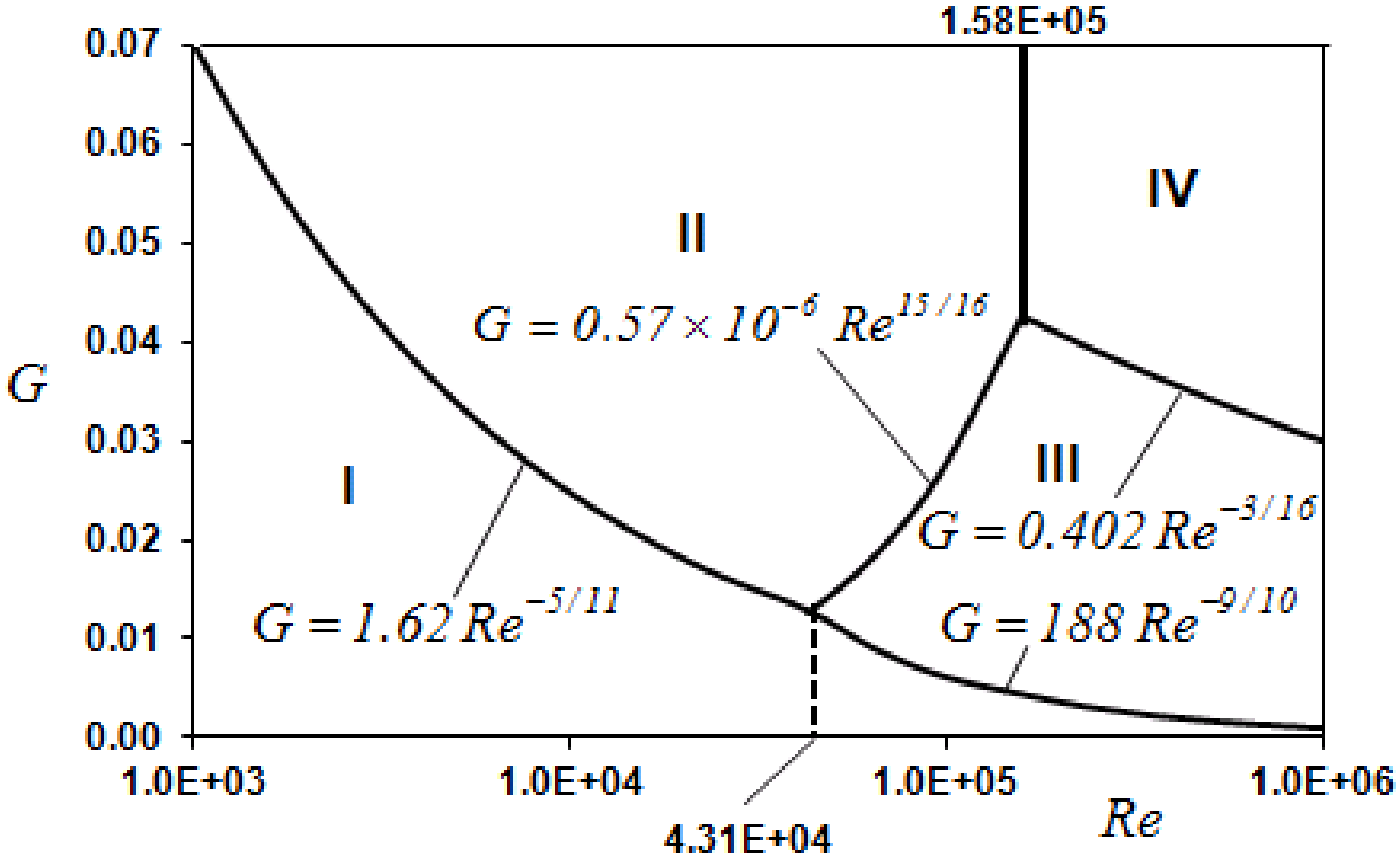
IV. Turbulent, separate boundary layers

$$\boxed{C_m = 0.051\, G^{1/10}\,\mathrm{Re}^{-1/5}}$$

$$\begin{cases} G > 0.402\,\mathrm{Re}^{-3/16} \\ \mathrm{Re} > 1.58\;10^{5} \end{cases}$$

Windage and Swirl Modeling in a General Cavity (6)

- Daily and Nece (1960)



Windage and Swirl Modeling in a General Cavity (7)

- 1987 Reference for Windage Calculation

Windage Rise and Flowpath Gas Ingestion in Turbine Rim Cavities

FRED HAASER
JAMES JACK
WILLIAM MCGREEHAN
General Electric Company, Cincinnati, Ohio

87-GT-164

ABSTRACT

Typically cooling air must be metered into cavities bordering turbine disks to offset cavity air temperature rise due to windage generated by air drag from rotating and stationary surfaces and the ingestion of hot mainstream gas. Being able to accurately estimate the minimum amount of cooling air required to purge turbine rim cavities is important towards providing optimum turbine cycle performance and hardware durability. Presented is an overview of a method used to model windage rise and ingestion on a macroscopic scale. Comparisons of model results to engine test data are included.

ACKNOWLEDGMENTS

The work reported in this paper has been funded by the General Electric Company, Aircraft Engines Business Group, in Cincinnati, Ohio where the three authors are currently employed in the analysis of secondary air (cooling) systems for turbofan engines in the 20,000 to 30,000 lb_f (8.9×10^4 to 13.3×10^4 N) thrust class range. The authors would like to extend credit to H. P. Rieck and R. W. Brown, also of General Electric, for their derivation of the wheel space modelling computer program, without which the results reported in this paper could not have been correlated.

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Windage and Swirl Modeling in a General Cavity (8)

- 1987 Reference for Windage Calculation 87-GT-164

Note: The methodology presented in this paper remains the industry bench-mark for windage computation.

Once the rotor drag and rotor bolt drag is known by the resolution of K (Appendix I), the cavity temperature rise due to windage can be calculated:

$$T_W = \frac{(M_R + M_{RB})}{(778)(C_p)(W_{GI} + W_{CI})} \quad (4)$$

- Correct form of Equation (4)

$$\Delta T_W = \frac{(M_R + M_{RB})\Omega}{(778)(C_p)(W_{GI} + W_{CI})}$$

Windage and Swirl Modeling in a General Cavity (9)

- **1987 Reference for Windage Calculation** 87-GT-164

Effects Increasing Angular Momentum and K
- o Rotor surface drag (disk pumping)
- o Rotor screw head drag
- o Coolant injected with tangential velocity same direction as rotor spin
- o Ingestion flow with tangential velocity same direction as rotor spin
- o Drag due to rotor surface imperfections such as instrumentation wires

Effects Decreasing Angular Momentum and K
- o Coolant injected axially or in tangential direction opposite to rotor spin
- o Inner radius flow extraction
- o Stator surface drag
- o Stator screw head drag
- o Outer radius flow extraction and misc. leakages
- o Drag due to stator surface imperfections such as instrumentation wires

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Windage and Swirl Modeling in a General Cavity (10)

1987 Reference for Windage Calculation

87-GT-164

Rotor Surface Drag

For disk surface drag Daily & Nece (3) derived:

$$M_R = (C_M)(.5\varrho\omega^2 a^5) \quad (6)$$

where C_M is the moment coefficient and Eq. 6 is for both faces of the disk. Eq. 6 was converted to the more generalized form:

$$M_R = (C_{FR}/g)(.5\varrho\omega^2 a^2 \int rds) \quad (7)$$

Where C_{FR} is the rotor surface friction coefficient, and the integral is the sum of moment-weighted rotor surface drag areas for only the side of the disk of interest. Through test experience, and the experimental data of Daily & Nece (3), C_{FR} is taken as:

$$C_{FR} = .042(1-K)^{1.35}/Re^{.2} \quad (8)$$

with the Reynolds number modified as:

$$Re = \varrho\omega a(a - R_I)/\mu \quad (9)$$

where a and R_I are the cavity's outer and inner radius, respectively.

Stator Surface Drag

For loss of angular momentum due to stator surface drag, Rieck uses the generalized form:

$$M_S = (C_{FS}/g)(.5\varrho\omega^2 a^2 \int rds) \quad (15)$$

where through the experimental data of Daily & Nece (3) and test experience the stator coefficient of friction is taken as:

$$C_{FS} = .063(K)^{1.87}/Re^{.2} \quad (16)$$

with Reynolds number as in Eq. 9. The integral in Eq. 15 includes the endwall static surface area thereby accounting for (s/a) drag as discussed by Daily & Nece.

Note: Equations (7) and (15) suggest the assumption of a uniform shear stress on the disk surface!

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Windage and Swirl Modeling in a General Cavity (11)

- 1987 Reference for Windage Calculation 87-GT-164

Note: Equations (7) and (15) suggest the assumption of a uniform shear stress on the disk surface!

$$\Gamma_{disk} = \int_0^{R_o} r\,\tau\,(2\pi r\,dr)$$

- Assume uniform tangential shear stress (τ_0)

$$\Gamma_{disk} = \tau_0 \int_0^{R_o} r\,(2\pi r\,dr) = \left(\frac{2\pi}{3}\right)\tau_0 R_0^3$$

$$\tau_0 = \frac{1}{2}\rho \overline{C_f}\,(R_0\,\Omega)^2$$

$$\Gamma_{disk} = \frac{1}{2}\rho C_m \Omega^2 R_o^5$$

$$C_m = \frac{2\pi}{3}\overline{C_f}$$

Windage and Swirl Modeling in a General Cavity (12)

1987 Reference for Windage Calculation 87-GT-164

Some Remarks

- The assumption of a constant τ_0 for a vertical disk is generally not true. In fact, the integral boundary layer analysis reveals that

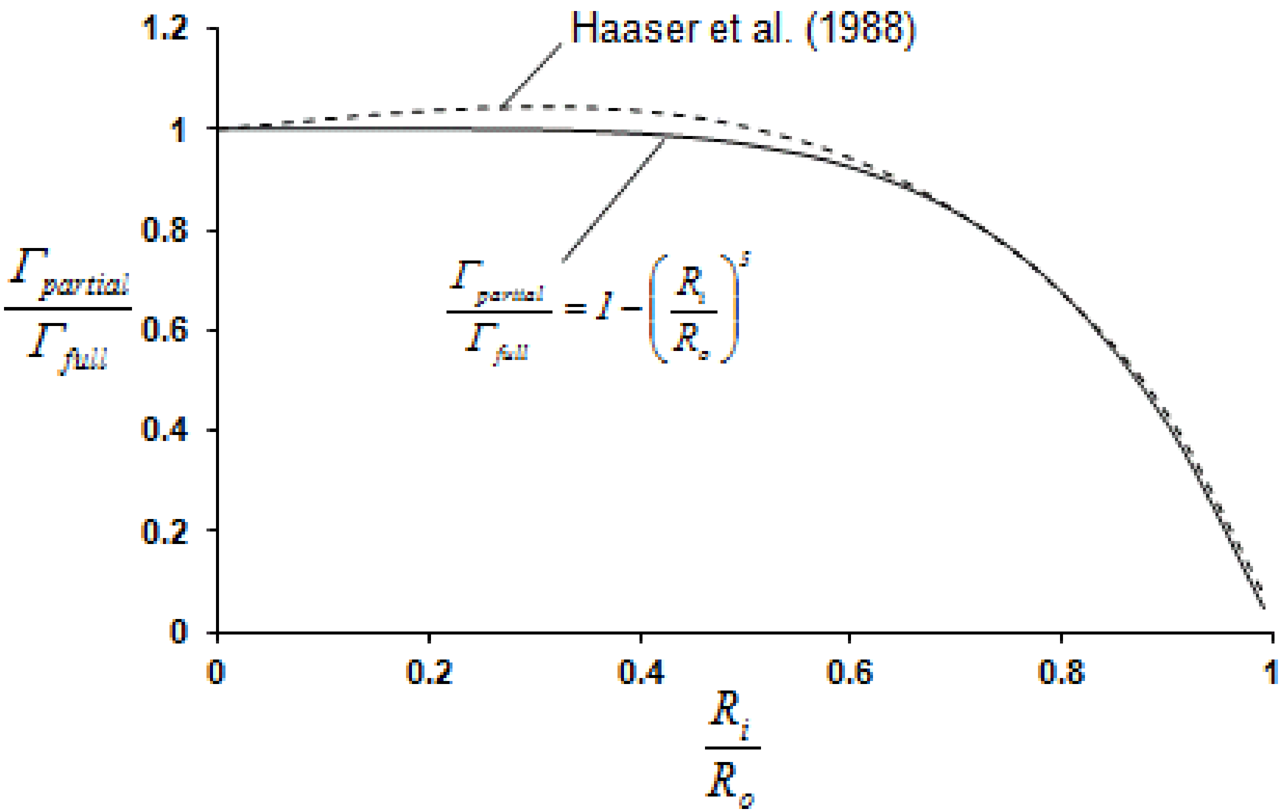
$$\tau \propto r^{1.6} \quad \Rightarrow \quad C_m = \frac{2\pi}{5} C_f$$

- The relation between C_M and $\overline{C}_f$ without any dependence on disk geometry is possible only for a full disk, i.e., disk with $R_1 = 0$.

- Both C_M and C_f are functions of radius which appears in $\mathrm{Re}^{-0.2}$.

Windage and Swirl Modeling in a General Cavity (13)

- Variation of disk torque ratio with radius ratio



Windage and Swirl Modeling in a General Cavity (14)

Single Rotor Cavity: Suggested Empirical Equations

- **Rotor Surface**

$$\boxed{\mathbf{C}_{\mathbf{f}_{\mathbf{R}}} = 0.070(1-\mathbf{S}_{\mathbf{f}})^{-0.65}\,\mathbf{Re}^{-0.2}}$$

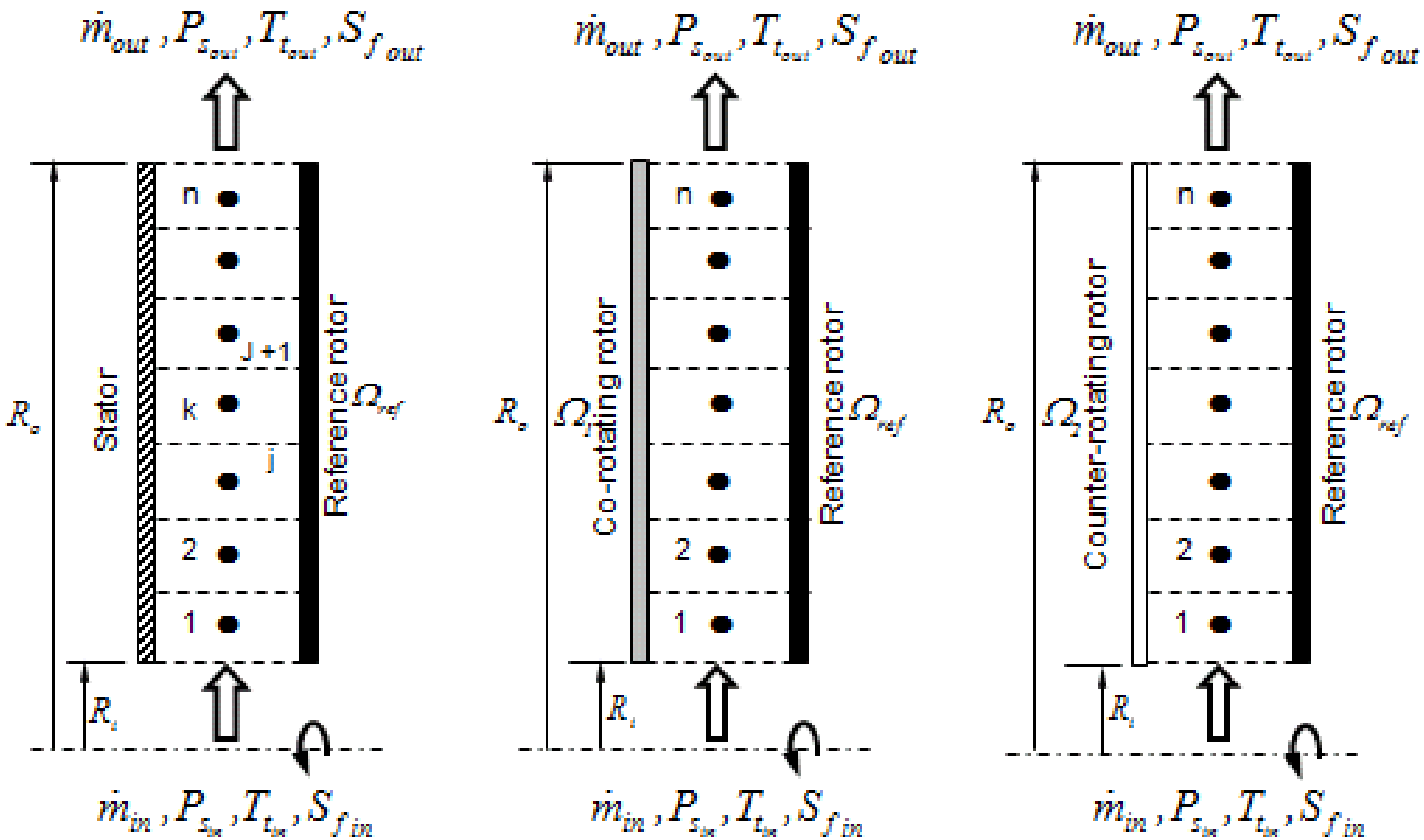
  - **Dynamic pressure:** $\boxed{0.5\rho(1-\mathbf{S}_{\mathbf{f}})^2\Omega^2\mathbf{r}^2}$

- **Stator Surface**

$$\boxed{\mathbf{C}_{\mathbf{f}_{\mathbf{S}}} = 0.105\mathbf{S}_{\mathbf{f}}^{-0.13}\,\mathbf{Re}^{-0.2}}$$

  - **Dynamic pressure:** $\boxed{0.5\rho\mathbf{S}_{\mathbf{f}}^2\Omega^2\mathbf{r}^2}$

Windage and Swirl Modeling in a General Cavity (15)



Windage and Swirl Modeling in a General Cavity (16)

Angular Momentum Balance – The Transcendental Equation

- For the control volume **k** whose inlet surface is designated by **j** and the outlet surface by **j+1**, we write the following angular momentum equation:

$$\boxed{\Gamma_{R_k} - \Gamma_{S_k} = \dot{m}(r_{j+1}V_{\theta_{j+1}} - r_j V_{\theta_j}) = \dot{m}(r_{j+1}^2 S_{f_{j+1}} - r_j^2 S_{f_j})\Omega_{ref}}$$

- Assuming a forced vortex core with swirl factor S_{f_k} such that $S_{f_{j+1}} = S_{f_k}$ and substituting

$$\begin{aligned}\Gamma_{R_k} &= C_{f_R}\frac{1}{2}\rho(1 - S_{f_k})^2\Omega_{ref}^2\int_{r_j}^{r_{j+1}} 2\pi r^4 dr \\ &= 0.044\rho(1 - S_{f_k})^{1.35}\Omega_{ref}^2(r_{j+1}^5 - r_j^5)\,Re^{-0.2}\end{aligned}$$

Windage and Swirl Modeling in a General Cavity (17)

❑ Angular Momentum Balance – The Transcendental Equation

and

$$\boxed{\Gamma_{S_k} = C_{f_S} \frac{1}{2} \rho S_{f_k}{}^2 \Omega_{ref}^2 \int_{r_j}^{r_{j+1}} 2\pi r^4 dr = 0.066 \rho S_{f_k}{}^{1.87} \Omega_{ref}^2 (r_{j+1}^5 - r_j^5) \mathrm{Re}^{-0.2}}$$

yields:

$$\boxed{\begin{gathered}\left\{0.044\rho(1-S_{f_k})^{1.35} - 0.066\rho S_{f_k}{}^{1.87}\right\} \Omega_{ref}^2 (r_{j+1}^5 - r_j^5) \left(\frac{\rho R_o^2 \Omega_{ref}}{\mu}\right)^{-0.2} \\ = \dot{m}(r_{j+1}^2 S_{f_k} - r_j^2 S_{f_j}) \Omega_{ref}\end{gathered}}$$

➢ Solve the above transcendental equation using the regula falsi method.

Windage and Swirl Modeling in a General Cavity (18)

- Changes is Static Pressure and Total Temperature Over the Control Volume
  - Static Pressure Change

$$P_{s_{j+1}} - P_{s_j} = \bar{\rho}\left(S_{f_k}\Omega_{ref}\right)^2\left(\frac{r_{j+1}^2 - r_j^2}{2}\right)$$

  - Static Pressure Change

$$T_{t_{j+1}} - T_{t_j} = \frac{\Gamma_{R_k}\Omega_{ref}}{\dot{m}c_p}$$

Windage and Swirl Modeling in a General Cavity (19)

Multi-Rotor Cavity: Suggested Empirical Equations

Rotor Surface

$$\boxed{C_{f_R} = 0.070\,\mathrm{sign}(\beta - S_f)\left|\beta - S_f\right|^{-0.65}\left|\beta\right|^{0.65}\,\mathrm{Re}^{-0.2}}$$

where

$$\boxed{\beta = \frac{\Omega}{\Omega_{ref}}}\qquad \boxed{\mathrm{sign}(\beta - S_f) \equiv \text{Sign of the term } (\beta - S_f)}\qquad \boxed{\mathrm{Re} = \frac{\rho R^2 \left|\beta\right| \Omega_{ref}}{\mu}}$$

- Dynamic pressure: $\boxed{0.5\rho(\beta - S_f)^2\Omega_{ref}^2 r^2}$

Stator Surface

$$\boxed{C_{f_S} = 0.105\left|S_f\right|^{-0.13}\mathrm{Re}^{-0.2}}$$

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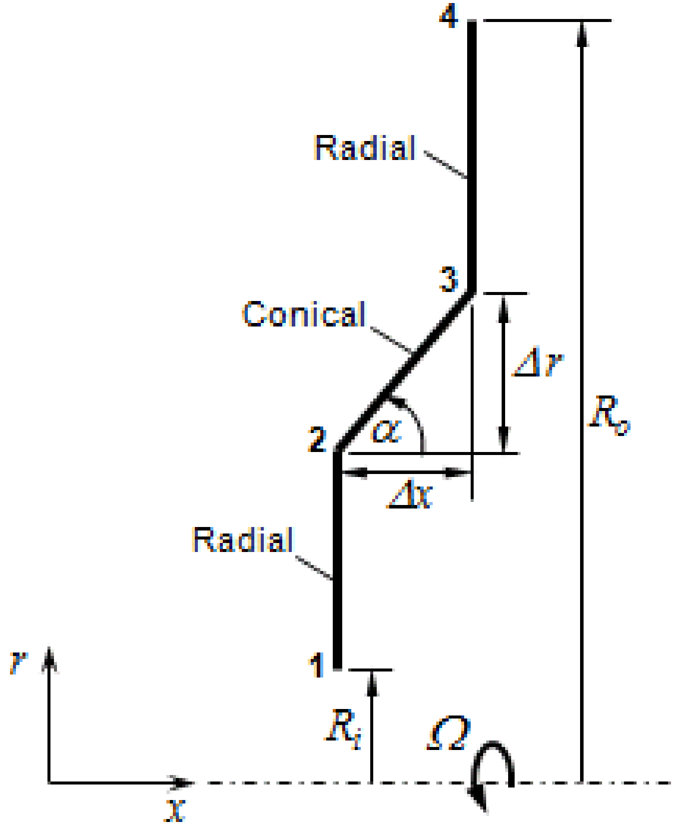
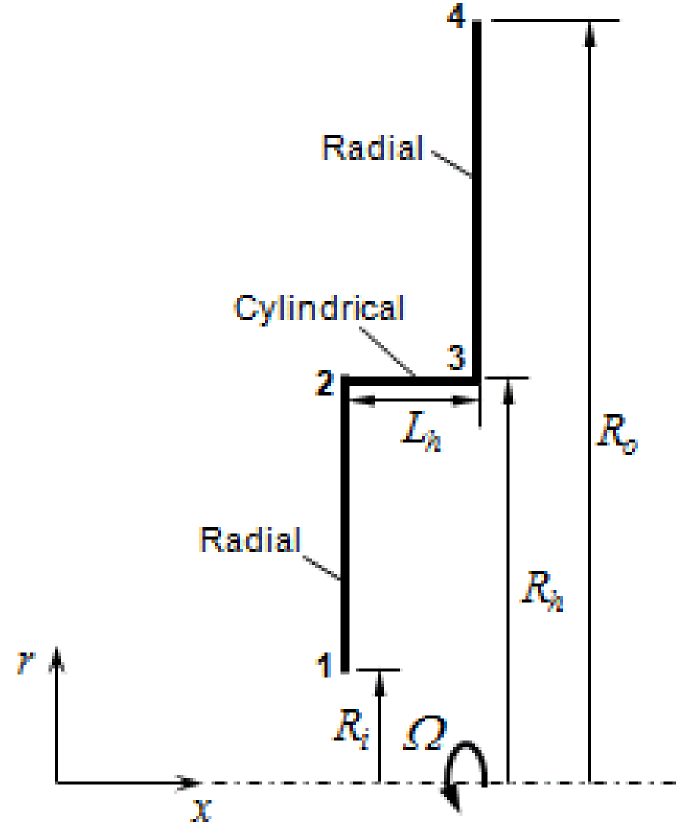
Windage and Swirl Modeling in a General Cavity (20)

- **Arbitrary Cavity Surface Orientation: Conical and Cylindrical Surfaces**





Windage and Swirl Modeling in a General Cavity (21)

- Arbitrary Cavity Surface Orientation: Conical and Cylindrical Surfaces
  - Conical Rotor Surface Segment

$$\Gamma_{R_{cone}} = C_{f_R} \frac{1}{2} \rho (\beta - S_f)^2 \Omega_{ref}^2 \int_{r_2}^{r_3} \frac{2\pi r^4}{\sin\alpha} dr$$

$$\Gamma_{R_{cone}} = \frac{0.044 \,\mathrm{sign}(\beta - S_f) \left|\beta - S_f\right|^{-0.65} \left|\beta\right|^{0.65} \rho (\beta - S_f)^2 \Omega_{ref}^2 (r_3^5 - r_2^5) \mathrm{Re}^{-0.2}}{\sin\alpha}$$

where

$$\sin\alpha = \sin(\tan^{-1}(\Delta r / \Delta x))$$

$$\mathrm{Re} = \frac{\rho R_o^2 \left|\beta\right| \Omega_{ref}}{\mu}$$

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Windage and Swirl Modeling in a General Cavity (22)

- **Arbitrary Cavity Surface Orientation: Conical and Cylindrical Surfaces**
  - **Conical Stator Surface Segment**

$$\boxed{\Gamma_{S_{cone}} = \frac{0.066\left|\mathbf{S_f}\right|^{-0.13}\rho \mathbf{S_f^2} \boldsymbol{\Omega}_{\mathbf{ref}}^2 (\mathbf{r_3^5} - \mathbf{r_2^5})\,\mathbf{Re}^{-0.2}}{\sin\alpha}}$$

where

$$\boxed{\mathbf{Re} = \frac{\rho \mathbf{R_o^2} \boldsymbol{\Omega}_{\mathbf{ref}}}{\mu}}$$

Windage and Swirl Modeling in a General Cavity (23)

- Arbitrary Cavity Surface Orientation: Conical and Cylindrical Surfaces
  - Cylindrical Rotor Surface Segment

$$\mathbf{C_{f_R}} = 0.042\,\mathbf{sign}(\boldsymbol{\beta} - \mathbf{S_f})\left|\boldsymbol{\beta} - \mathbf{S_f}\right|^{-0.65}\left|\boldsymbol{\beta}\right|^{0.65}\mathbf{Re}^{-0.2}$$

$$\mathbf{Re} = \frac{\rho \mathbf{R_h^2}\left|\boldsymbol{\beta}\right|\boldsymbol{\Omega}_{\mathbf{ref}}}{\mu}$$

- Torque:

$$\Gamma_{\mathbf{R_{cylinder}}} = \mathbf{C_{f_R}}\frac{1}{2}\rho(\boldsymbol{\beta} - \mathbf{S_f})^2\boldsymbol{\Omega}_{\mathbf{ref}}^2\mathbf{R_h^2}\int_{\mathbf{x_2}}^{\mathbf{x_3}} 2\pi\mathbf{R_h^2}\mathbf{dx}$$

$$\Gamma_{\mathbf{R_{cylinder}}} = 0.132\,\mathbf{sign}(\boldsymbol{\beta} - \mathbf{S_f})\left|\boldsymbol{\beta} - \mathbf{S_f}\right|^{-0.65}\left|\boldsymbol{\beta}\right|^{0.65}\rho(\boldsymbol{\beta} - \mathbf{S_f})^2\boldsymbol{\Omega}_{\mathbf{ref}}^2\mathbf{R_h^4}\mathbf{L_h}\,\mathbf{Re}^{-0.2}$$

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Windage and Swirl Modeling in a General Cavity (24)

- Arbitrary Cavity Surface Orientation: Conical and Cylindrical Surfaces
  - Cylindrical Stator Surface Segment

$$\boxed{C_{f_s} = 0.063\left|S_f\right|^{1.87}\,Re^{-0.2}}$$

$$\boxed{\Gamma_{S_{cylinder}} = 0.198\left|S_f\right|^{-0.13}\,\rho S_f^2 \Omega_{ref}^2 R_h^4 L_h\,Re^{-0.2}}$$

where

$$\boxed{Re = \frac{\rho R_h^2 \Omega_{ref}}{\mu}}$$

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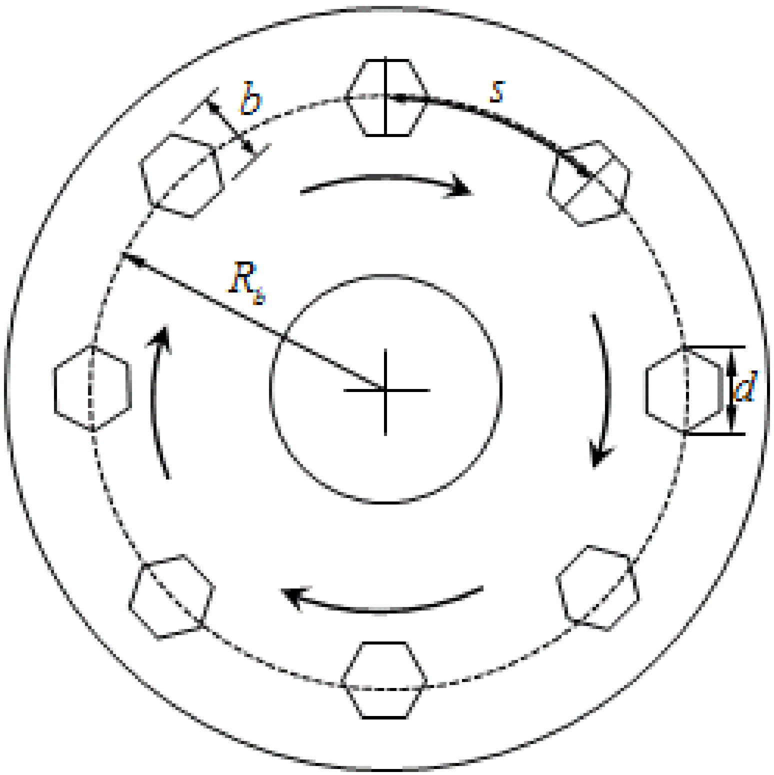
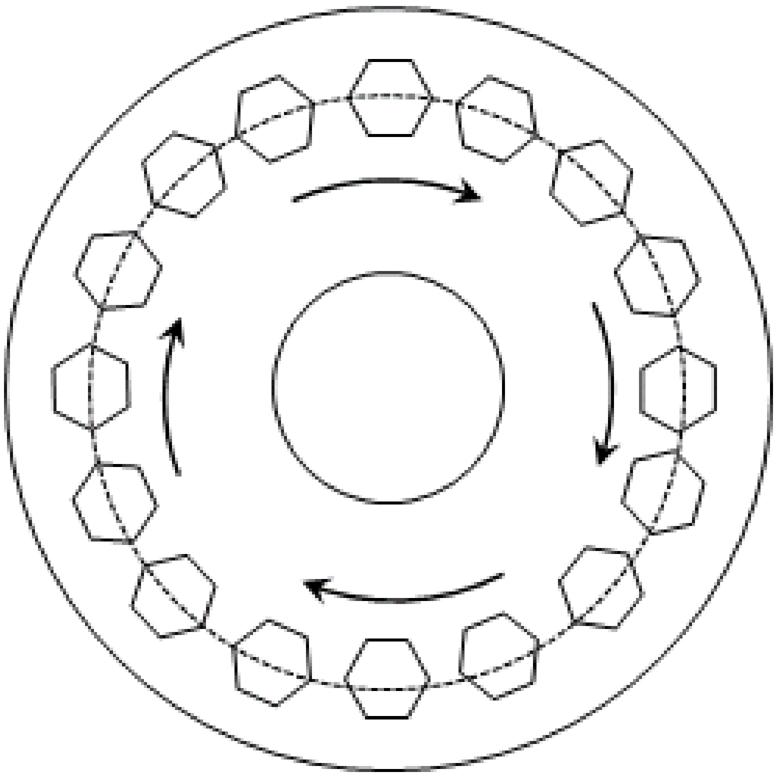
Windage and Swirl Modeling in a General Cavity (25)

❑ Bolts on Stator and Rotor Surfaces



❖ Small interference



❖ Large interference

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Windage and Swirl Modeling in a General Cavity (26)

Bolts on Stator and Rotor Surfaces

Bolts on Rotor Surface:

$$\Gamma_{R_b} = 0.5 N_b h b C_{D_b} I_b R_b^3 \rho \Omega_{ref}^2 (\beta - S_f)^2$$

Bolts on Stator Surface:

$$\Gamma_{S_b} = 0.5 N_b h b C_{D_b} I_b R_b^3 \rho \Omega_{ref}^2 S_f^2$$

where

$N_b \equiv$ Number of bolts

$h \equiv$ Bolt height from the disk surface

$b \equiv$ Bolt width along the radial direction

$C_{D_b} \equiv$ Baseline drag coefficient of each bolt (~ 0.6)

$R_b \equiv$ Bolts pitch circle diameter

$I_b \equiv$ Bolts interference factor

Windage and Swirl Modeling in a General Cavity (27)

Bolts on Stator and Rotor Surfaces

- Approximate correlations to compute interference bolts interference factor
  - For $1 \le \frac{s}{d} < 2$

$$\boxed{I_b = 20.908 - 61.855\left(\frac{s}{d}\right) + 66.616\left(\frac{s}{d}\right)^2 - 30.481\left(\frac{s}{d}\right)^3 + 5.051\left(\frac{s}{d}\right)^4}$$

  - For $1 \le \frac{s}{d} < 2$

$$\boxed{\begin{aligned} I_b = 5.7185 - 6.5982\left(\frac{s}{d}\right) + 3.1375\left(\frac{s}{d}\right)^2 - 0.6878\left(\frac{s}{d}\right)^3 \\ + 0.0717\left(\frac{s}{d}\right)^4 - 0.0029\left(\frac{s}{d}\right)^5 \end{aligned}}$$

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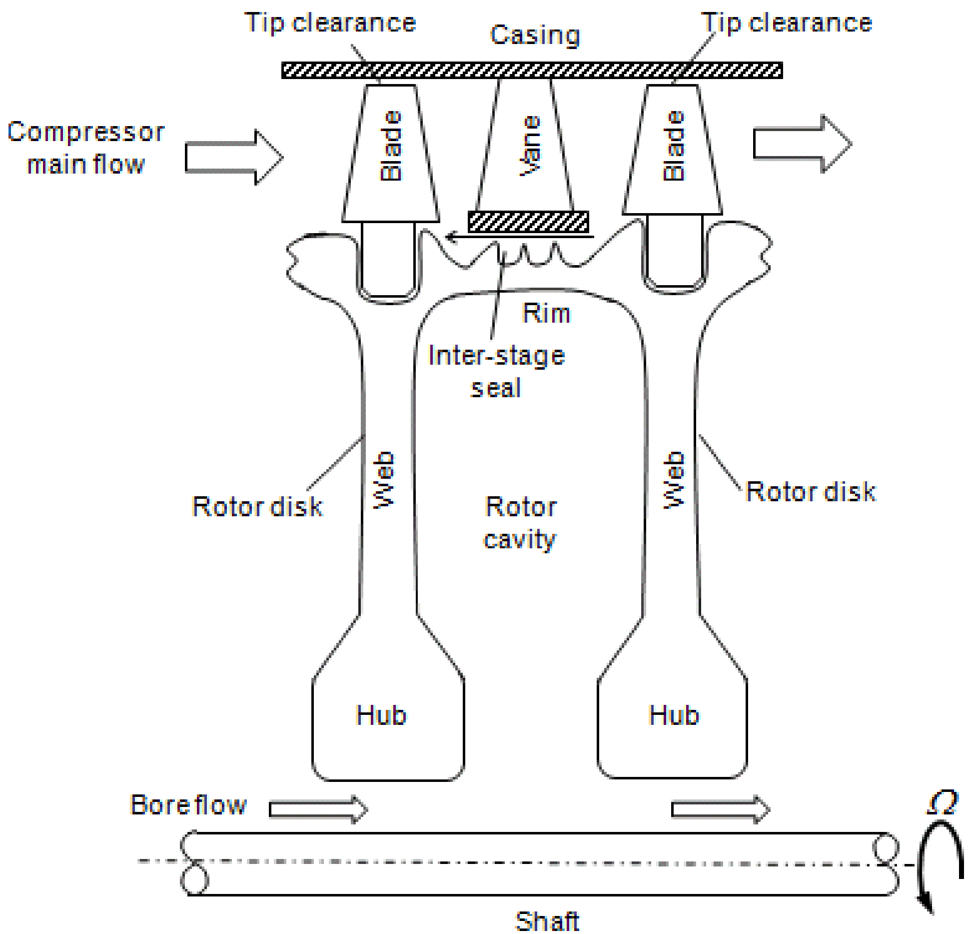
Windage and Swirl Modeling in a General Cavity (28)

Concluding Remarks

- **Don't use free vortex assumption in SAS modeling, instead, model rotor-rotor and rotor-stator cavity as a generalized vortex computed from stack-cavity analysis, with a forced vortex core in each sub-cavity.**
- **The primary role of the generalized vortex in SAS modeling is to accurately compute pressure changes in the rotor cavity.**
- **Don't use isentropic forced vortex temperature rise to compute adiabatic temperature changes in the rotor cavity, instead, use temperature changes due to windage.**
- **Make sure that the rotor cavity code is solution-robust before it's integrated into the SAS flow network analysis!**

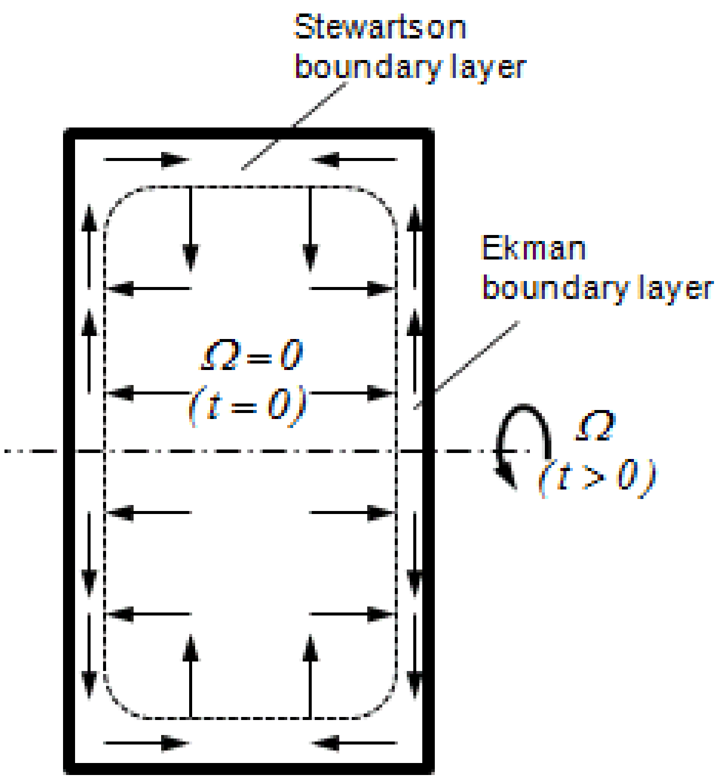
Compressor Rotor Cavity (1)

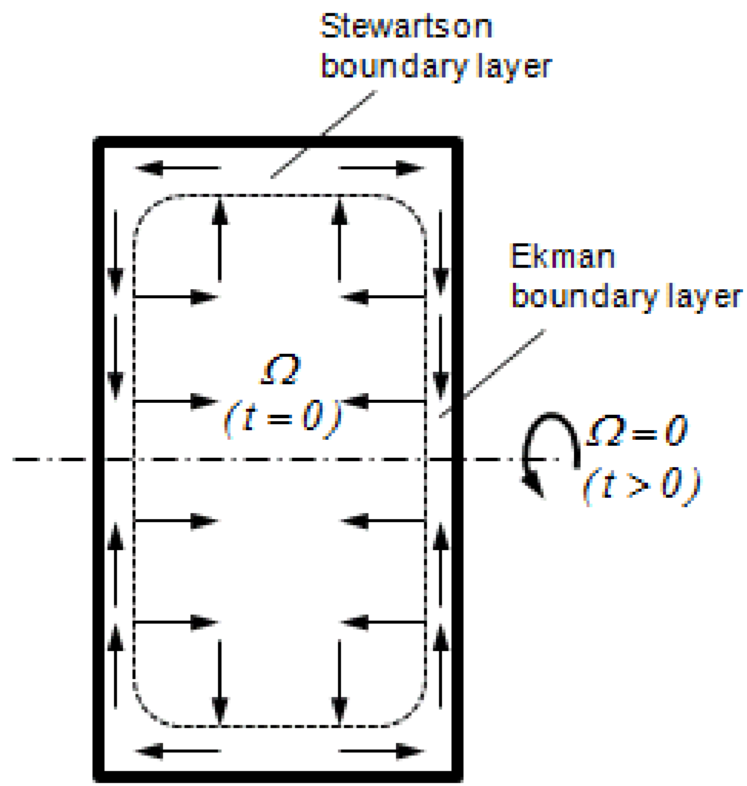


Compressor Rotor Cavity (2)

❑ Flow and Heat Transfer Physics

$$\boxed{dP_s / dr = \rho r \Omega_f^2}$$





- ❖ Spin-up from rest
- ❖ Spin-down to rest

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Compressor Rotor Cavity (3)

Heat Transfer Modeling with Bore Flow

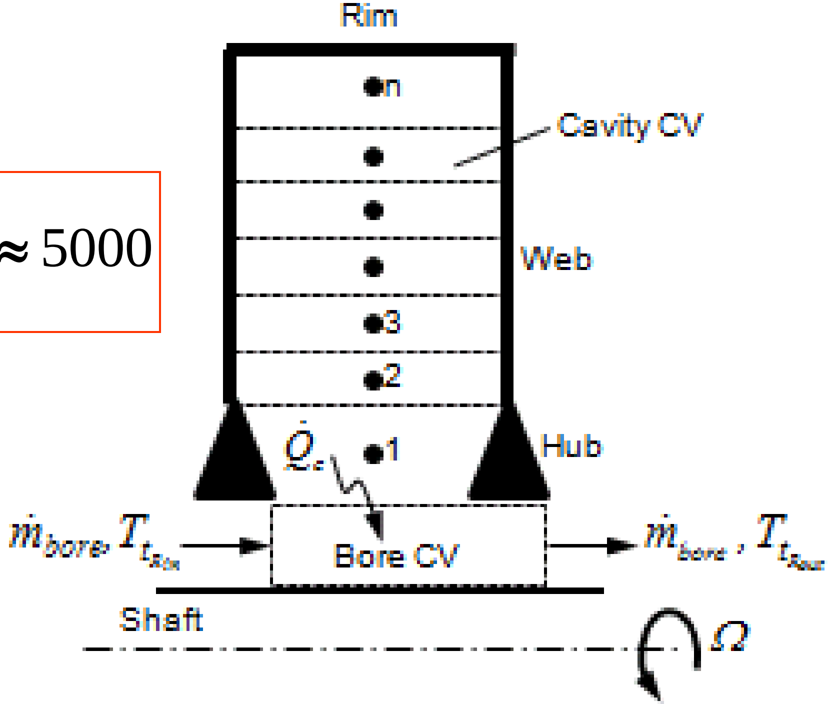
- **Centrifugally-driven buoyant convection (CDBC):**

$$\boxed{\mathbf{g_c} = \mathbf{r}\Omega^2}$$

- **Gravitationally-driven buoyant convection (GDBC):** $\boxed{\mathbf{g} = 9.81\,\mathbf{m/s^2}}$

At $\mathbf{r} = 0.5\,\mathbf{m}$ and $\Omega = 3000\,\mathbf{rpm}$: $\boxed{\dfrac{\mathbf{g_c}}{\mathbf{g}} \approx 5000}$

- **CDBC:**
  - Colder fluid flows radially outward.
  - Hotter fluid flows radially inward.



Compressor Rotor Cavity (4)

Heat Transfer Modeling with Bore Flow

Bore Control Volume

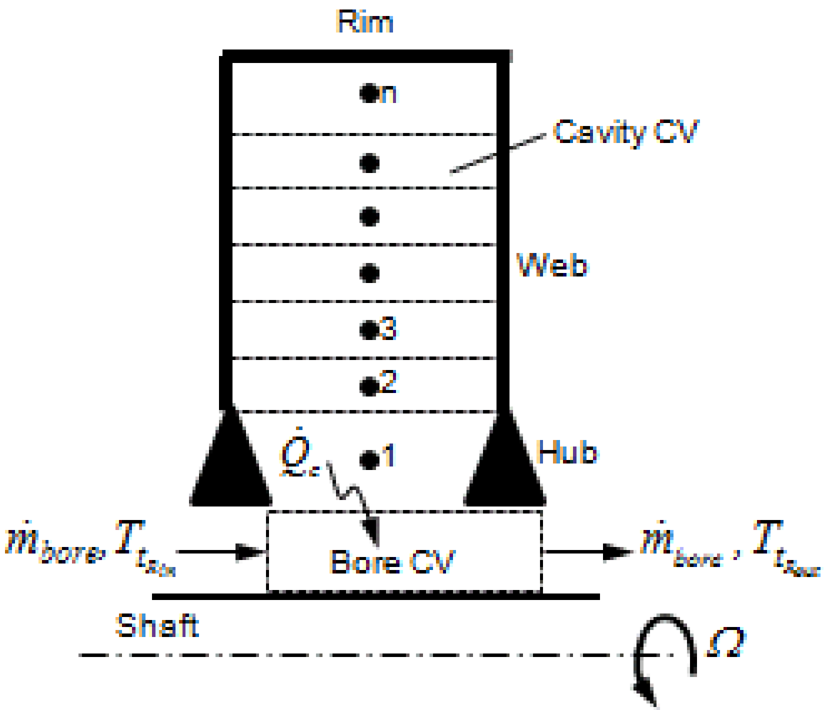
$$T_{t_{R_{out}}} = T_{t_{R_{in}}} + \frac{\dot{Q}_c}{\dot{m}_{bore} c_p}$$

Cavity Control Volume

$$\dot{q}_c = h\left(T_w - T_{aw}\right)$$

$$T_{aw} - \frac{r^2 \Omega^2}{2c_p} = T_{t_{R_{in}}} - \frac{r_{bore}^2 \Omega^2}{2c_p}$$

$$T_{aw} = T_{t_{R_{in}}} + \frac{\Omega^2}{2c_p}\left(r^2 - r_{bore}^2\right)$$



Compressor Rotor Cavity (5)

Heat Transfer Modeling with Bore Flow

$$\boxed{\mathrm{Nu} = \mathrm{C}(\mathrm{Ra})^{\mathrm{m}}}$$

Physical situation	*Ra* range	*C*	*m*
Vertical surface	$10^4 - 10^9$	0.59	$1/4$
Vertical surface	$10^9 - 10^{12}$	0.13	$1/3$
Horizontal surface ($T_w > T_{aw}$, CDBC)	$10^5 - 2\times 10^7$	0.54	$1/4$
Horizontal surface ($T_w > T_{aw}$, CDBC)	$2\times 10^7 - 3\times 10^{10}$	0.14	$1/3$
Horizontal surface ($T_w < T_{aw}$, CDBC)	$3\times 10^5 - 3\times 10^{10}$	0.27	$1/4$

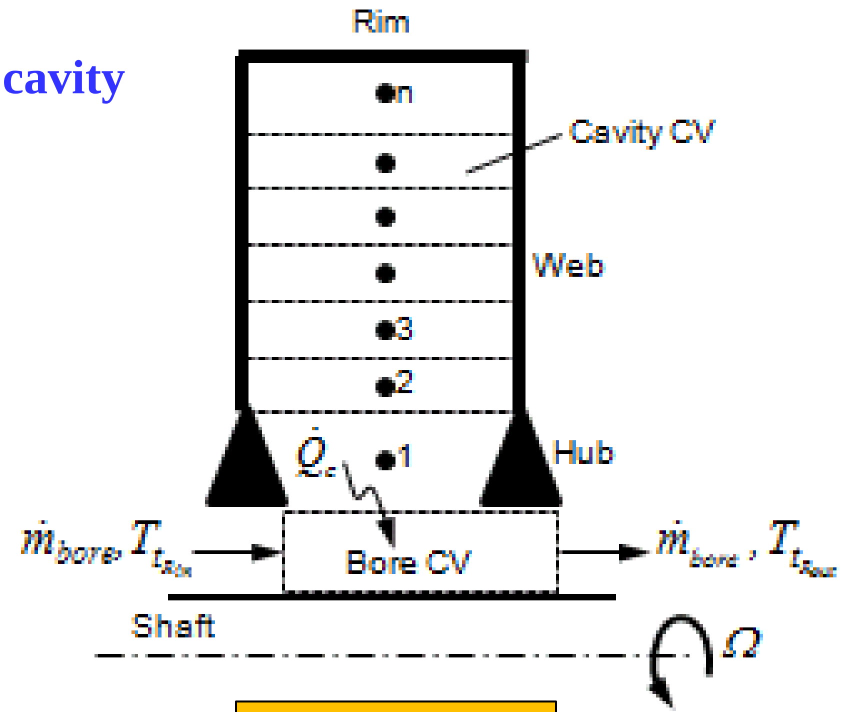
Compressor Rotor Cavity (6)

Heat Transfer Modeling of Closed Cavity

$\tilde{m} \equiv$ Constant mass of air in the closed cavity

$$\dot{\tilde{m}} = \frac{\tilde{m}}{\Delta t}$$

$$T_{t_R}(t+\Delta t) = T_{t_R}(t) + \frac{\dot{Q}_c}{\dot{\tilde{m}} c_p}$$



$$\dot{m}_{bore} = 0$$

QUESTIONS?

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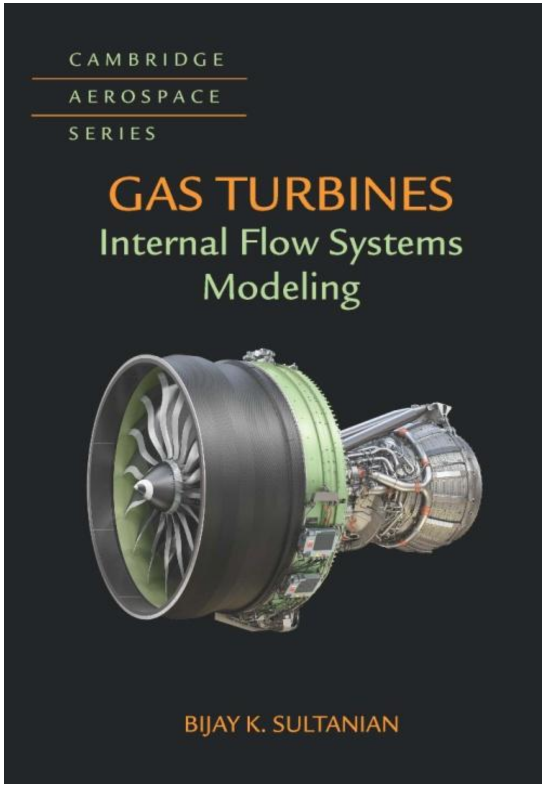
THANK YOU!



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Physics-Based Modeling of Gas Turbine Secondary Air Systems

Module 5: Physics-Based Modeling – Part II

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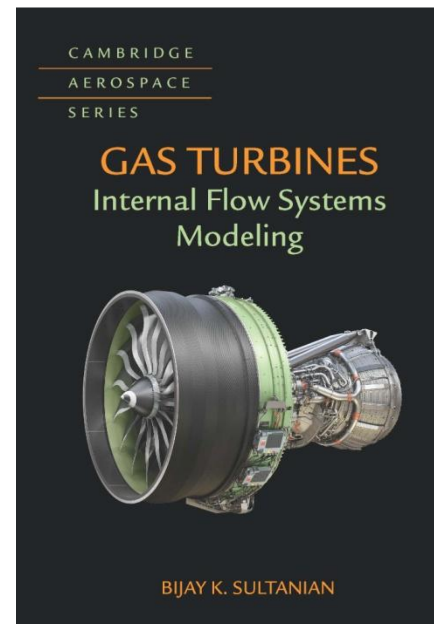

ASME 2019 TURBO EXPO

Phoenix, Arizona, USA
Sunday, June 16, 2019

Module 5

Physics-Based Modeling – Part II

Module 5

- **Physics-Based Modeling – Part II**
  - **Hot gas ingestion: ingress and egress**
  - **Single-orifice model**
  - **Multiple-orifice spoke model**

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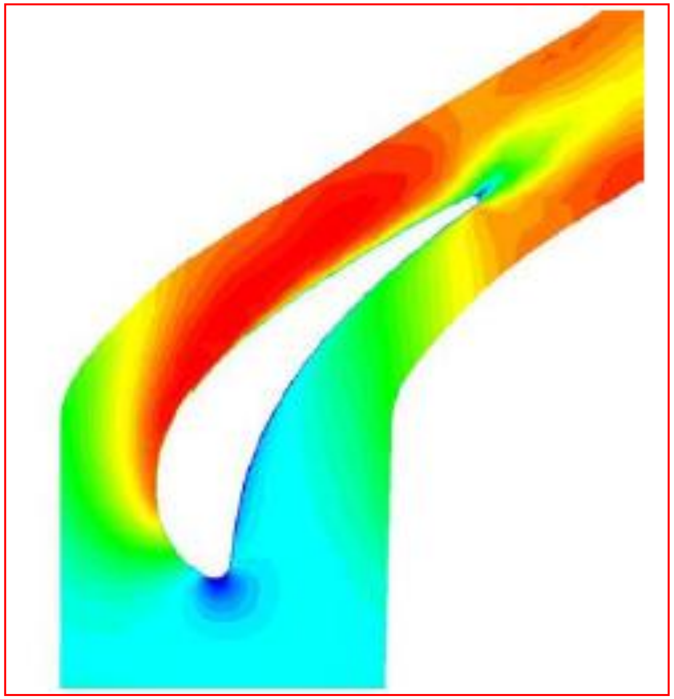
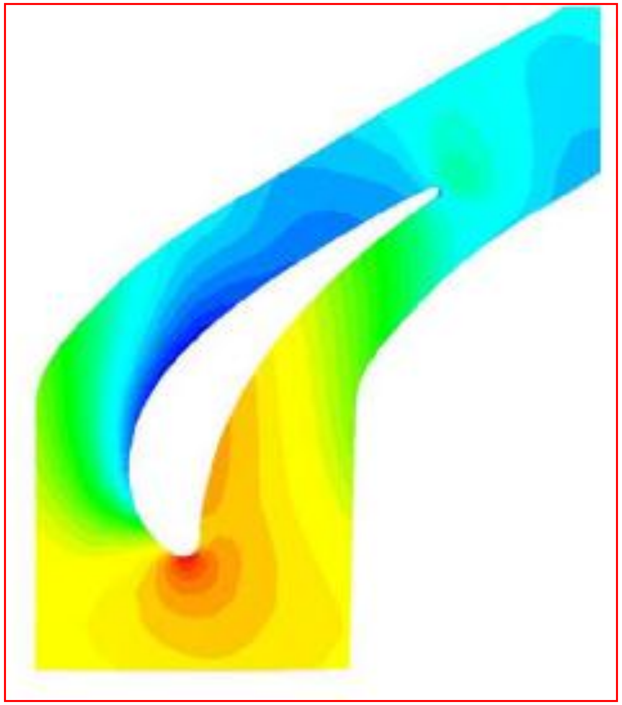
Hot Gas Ingestion: Ingress and Egress (1)

- **Physics of Hot Gas Ingestion**
  - **Asymmetry in the Turbine Gas Path**



Mach number distribution



Static pressure distribution

Hot Gas Ingestion: Ingress and Egress (2)

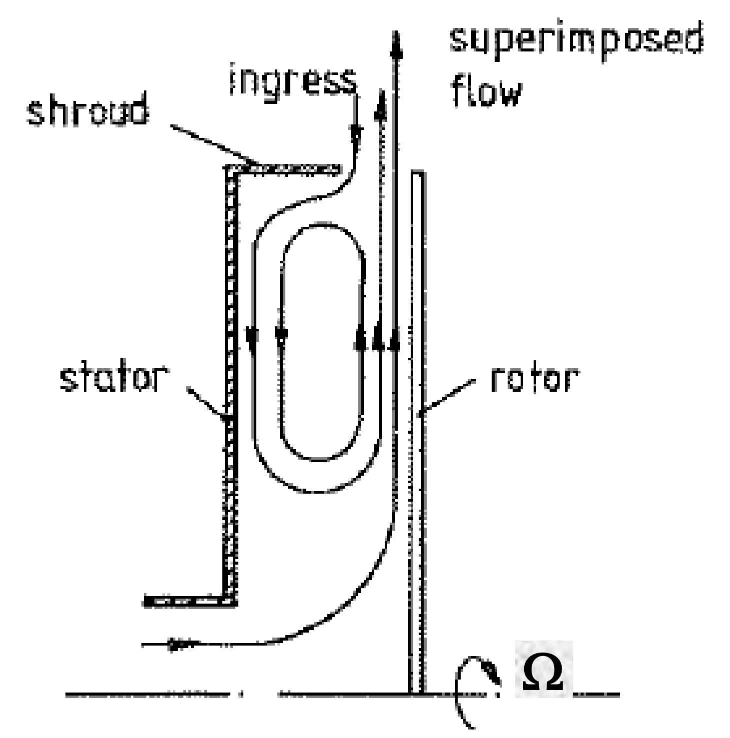
Physics of Hot Gas Ingestion

- Rotor Disk Pumping beneath a Forced Vortex

$$\dot{m}_{disk\ pump}(r) = 0.219\,\mu\,R\,Re_{\theta}^{0.8}\left(\frac{r}{R}\right)^{2.6}\zeta$$

$$\zeta = (1 - 0.51\,S_f)(1 - S_f)^{1.6}$$

$S_f \equiv$ Swirl factor



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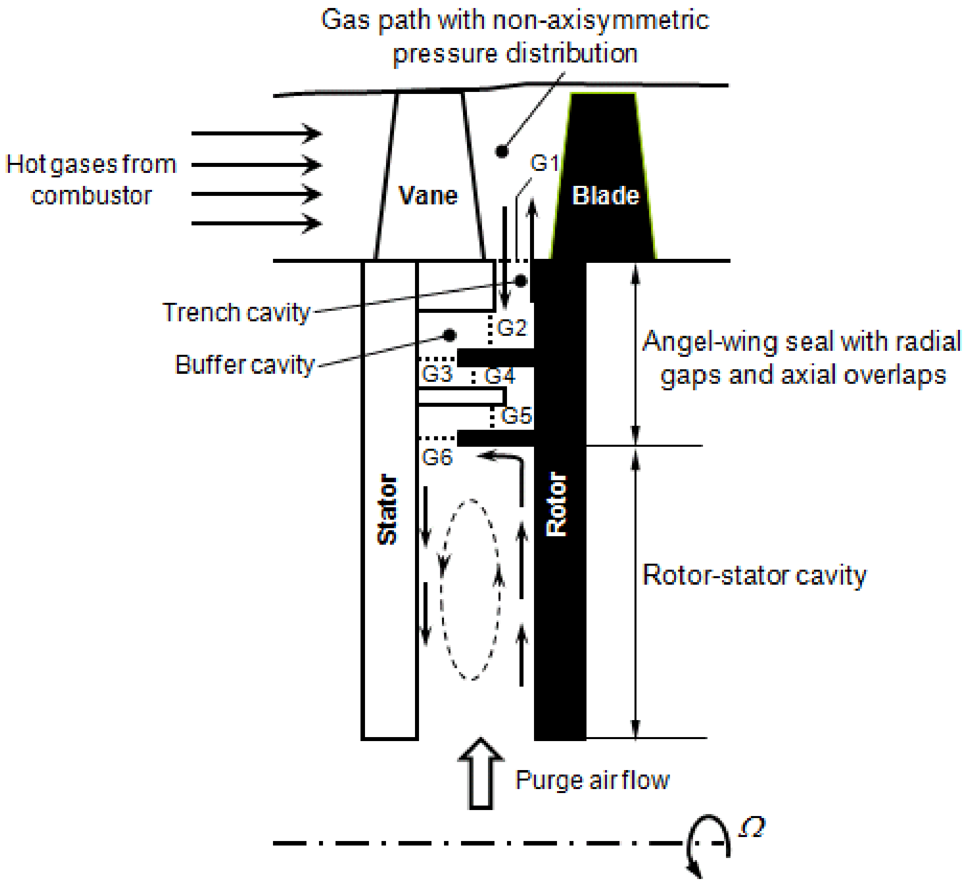
Hot Gas Ingestion: Ingress and Egress (3)

❑ Physics of Hot Gas Ingestion



Hot Gas Ingestion: Ingress and Egress (4)

Physics of Hot Gas Ingestion

- **Primary Factors**
  - Periodic vane/blade pressure field (non-axisymmetric pressure distribution in the main flowpath of hot gases)
  - Disk pumping in the rotor-stator cavity
  - Rim seal geometry (radial and axial clearances and overlaps)
  - Purge sealing and cooling air flow rate

- **Secondary Factors**
  - Unsteadiness in 3-D flow field
  - Pressure fluctuations in the wheel space
  - Turbulent transport in the platform and outer cavity region

Hot Gas Ingestion: Ingress and Egress (5)

Design Strategy

- Establish the minimum cavity purge flow needed for acceptable windage temperature rise and heat transfer in the rotor-stator cavity.
- Establish the gas path asymmetric pressure boundary conditions from an appropriate CFD solution.
- Design a seal that will limit the ingress (hot gas ingestion) to trench (the first design target) and buffer cavities (the second design target if we can't meet the first).

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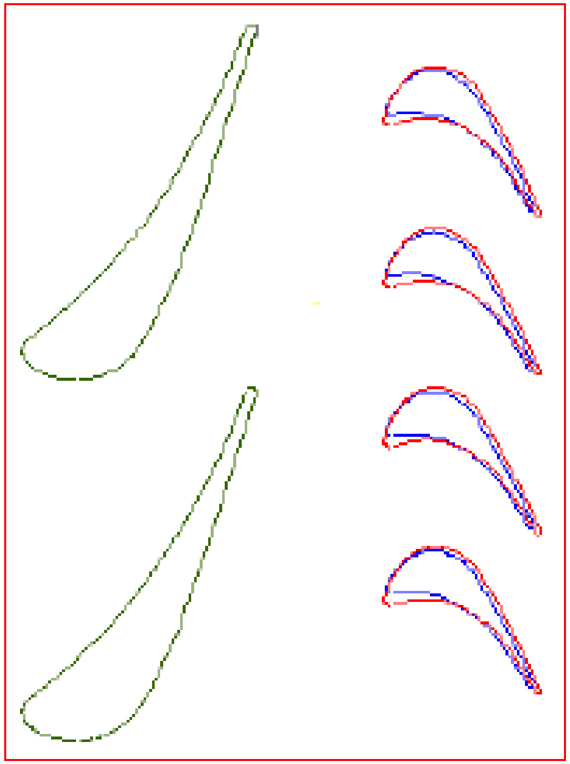
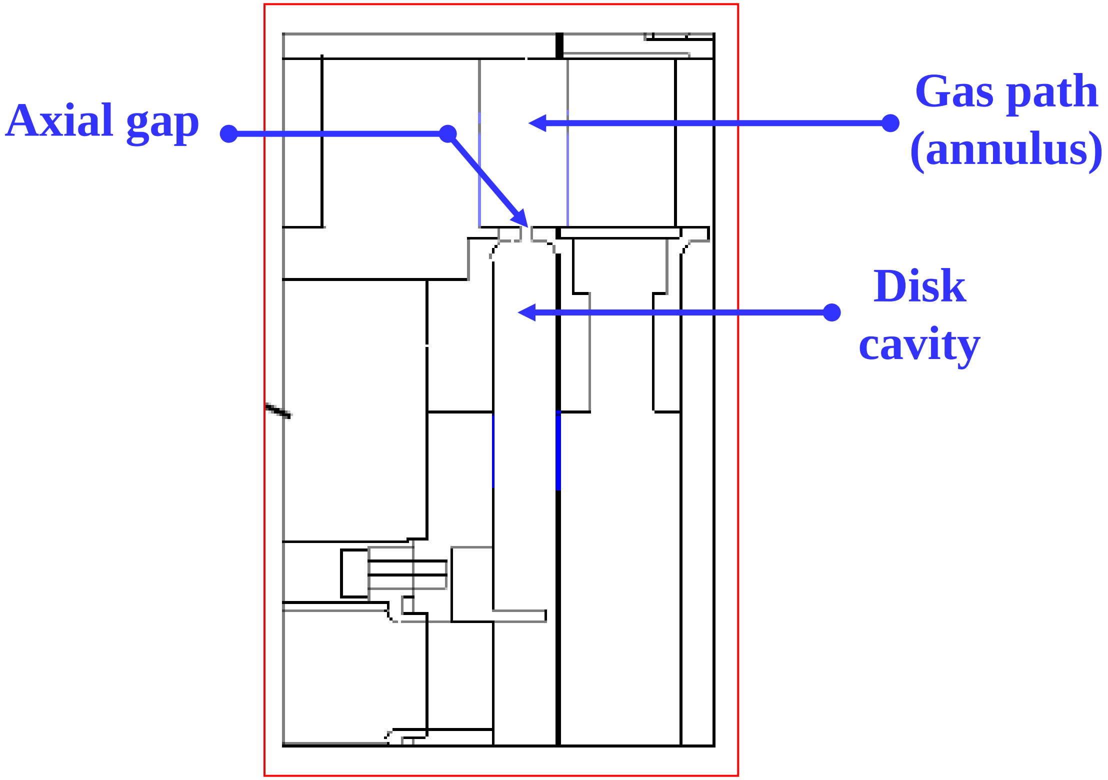
Single-Orifice Model (1)

❑ Rim Seal Geometry

❖ Scanlon et al. (2004)

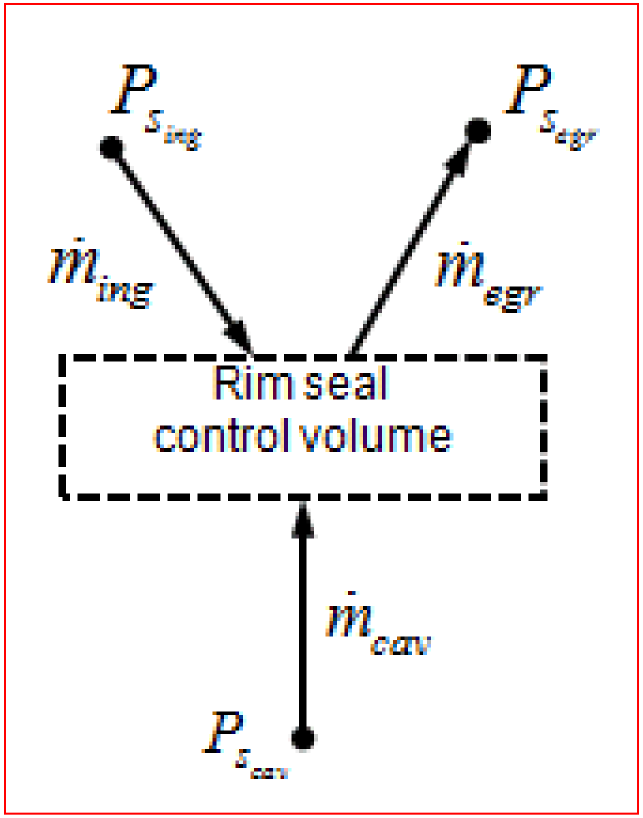




Single-Orifice Model (2)

- Rim Seal Control Volume

- Scanlon et al. (2004)



- Mass Conservation (Continuity Equation)

$$\boxed{\dot{m}_{egr} = \dot{m}_{ing} + \dot{m}_{cav}}$$

- Energy Balance (Constant c_p)

$$\boxed{T_{t_{egr}} = \frac{\dot{m}_{ing} T_{t_{ing}} + \dot{m}_{cav} T_{t_{cav}}}{\dot{m}_{egr}}}$$

Single-Orifice Model (3)

Assumptions

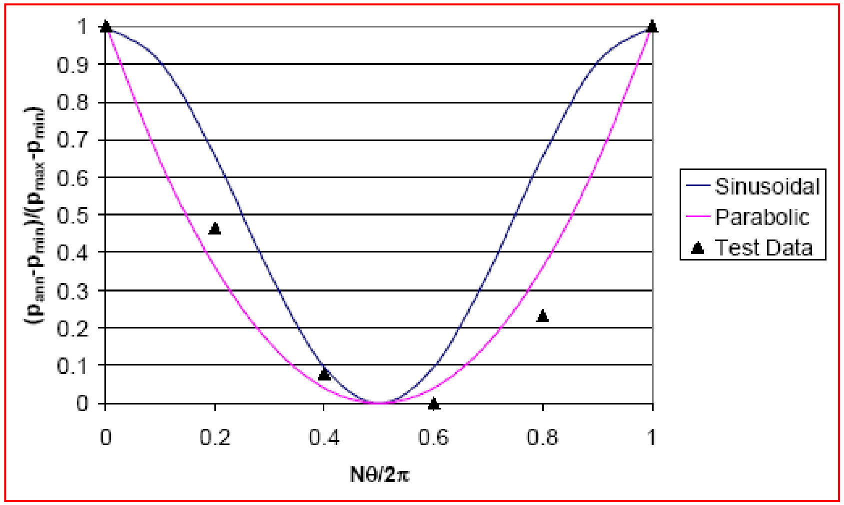
❖ Scanlon et al. (2004)

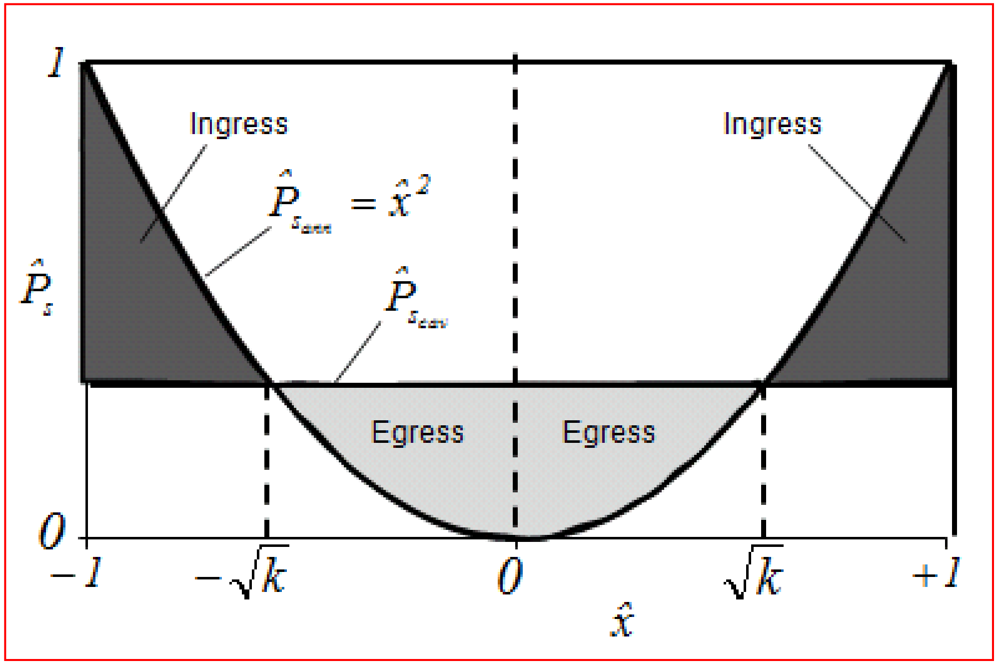
- For calculating $\dot{m}_{ing}$ and $\dot{m}_{egr}$ across the rim seal gap of total area A_{gap}, we make the following assumptions:
  - Incompressible flow with constant density ρ.
  - Plenum conditions prevail on either side of the rim seal gap.
  - Axisymmetric distribution of static pressure $P_{s_{cav}}$ in the wheel-space at the rim seal gap; for the egress flow, this pressure acts as the total pressure.
  - Parabolic distribution of static pressure in the gaspath annulus at the rim seal gap; for the ingress flow, this pressure acts as the total pressure.

Single-Orifice Model (4)

❑ Gaspath Pressure Variation

❖ **Scanlon et al. (2004)**

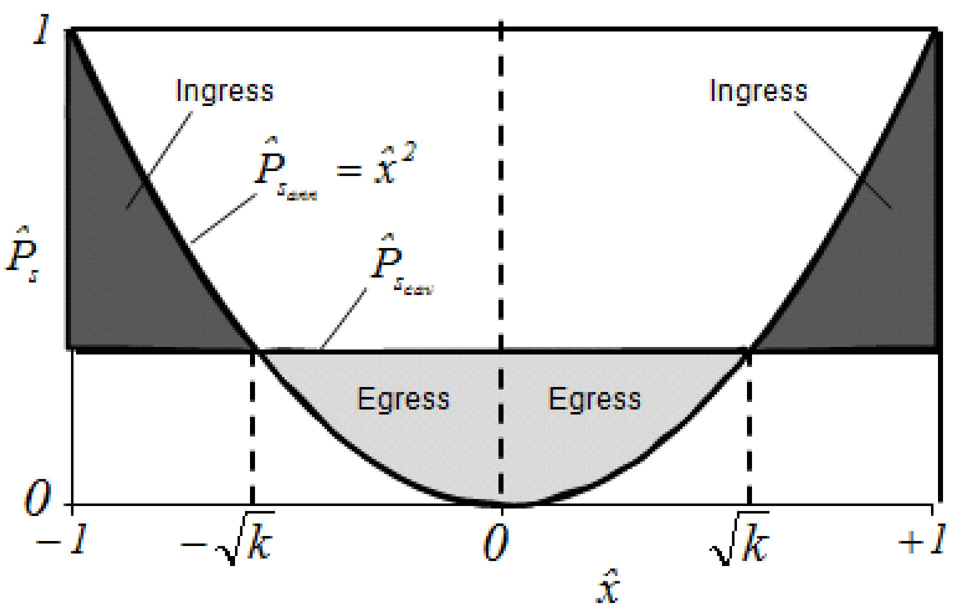




Single-Orifice Model (5)

❑ Gaspath Pressure Variation

❖ Scanlon et al. (2004)



$P_{s_{max}} \equiv$ Maximum static pressure in the profile

$P_{s_{min}} \equiv$ Minimum static pressure in the profile

- Define a dimensionless static pressure:

$$\hat{P}_s = (P_s - P_{s_{min}}) / (P_{s_{max}} - P_{s_{min}})$$

- Annulus static pressure:

$$\hat{P}_{s_{ann}} = (P_{s_{ann}} - P_{s_{min}}) / (P_{s_{max}} - P_{s_{min}})$$

- Cavity static pressure:

$$\hat{P}_{s_{cav}} = (P_{s_{cav}} - P_{s_{min}}) / (P_{s_{max}} - P_{s_{min}})$$

Single-Orifice Model (6)

Mass Conservation at Rim Seal

Scanlon et al. (2004)

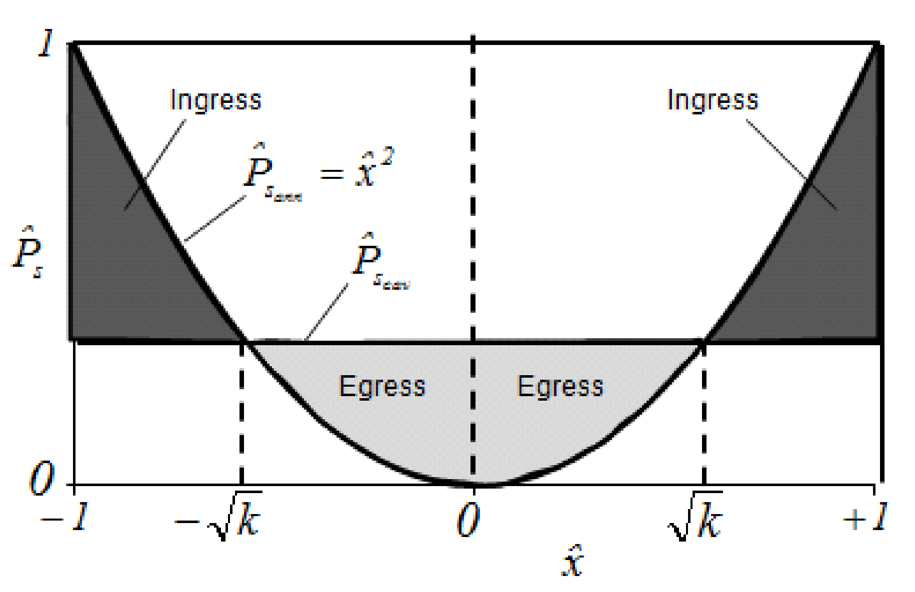
$\mathbf{N} \equiv$ **Number of vanes in the flowpath**

$\mathbf{N/2\pi} \equiv$ **Annular sector per vane**

$\mathbf{A_{gap}} \equiv$ **Total flow area of rim seal gap**

$\mathbf{\hat{A}_{gap} = A_{gap}/N} \equiv$ **Seal area per vane**

$$\boxed{\hat{x} = \frac{N\theta}{\pi} - 1, \; 0 \le \theta \le \frac{2\pi}{N}}$$



Note: When θ varies from 0 to $N/2\pi$, $\hat{x}$ varies from -1 to +1.

Ingress: $\hat{P}_{s_{ann}} > \hat{P}_{s_{cav}}$ **Egress:** $\hat{P}_{s_{cav}} > \hat{P}_{s_{ann}}$

Single-Orifice Model (7)

❑ Mass Conservation at Rim Seal

❖ Scanlon et al. (2004)

The points of intersection between $\hat{P}_{s_{cav}}$ and $\hat{P}_{s_{ann}}$ correspond to $\hat{x} = -\sqrt{k}$ and $\hat{x} = \sqrt{k}$ where $k = \hat{P}_{s_{cav}}$.

$$d\hat{A}_{gap} = \frac{N\hat{A}_{gap}d\theta}{2\pi} \quad \text{where} \quad 0 \le \theta \le \frac{2\pi}{N}$$

Since $\hat{x} = \dfrac{N\theta}{\pi} - 1$, $\quad d\hat{x} = \dfrac{Nd\theta}{\pi}$ or $d\theta = \dfrac{\pi d\hat{x}}{N}$

Thus

$$\boxed{d\hat{A}_{gap} = \frac{N\hat{A}_{gap}}{2\pi}\left(\frac{\pi d\hat{x}}{N}\right)} \quad \text{or} \quad \boxed{d\hat{A}_{gap} = \frac{\hat{A}_{gap}d\hat{x}}{2}}$$

Single-Orifice Model (8)

- ❖ Scanlon et al. (2004)

❑ Mass Conservation at Rim Seal

- ❖ Ingress mass flow rate $\dot{m}_{ing}$

Assuming one-dimensional incompressible flows at the rim seal, we can write

$$\int d\dot{m}_{ing} = C_d\sqrt{2\rho}\int\left(\sqrt{(P_{s_{ann}} - P_{s_{cav}})}\right)dA_{gap} = NC_d\sqrt{2\rho}\int\left(\sqrt{(P_{s_{ann}} - P_{s_{cav}})}\right)d\hat{A}_{gap}$$

or

$$\dot{m}_{ing} = NC_d\sqrt{2\rho}\int\left(\sqrt{(P_{s_{ann}} - P_{s_{cav}})}\right)d\hat{A}_{gap}$$

$$= 2N\left(\frac{\hat{A}_{gap}}{2}\right)C_d\sqrt{2\rho}\int_{\sqrt{k}}^{1}\left(\sqrt{(P_{s_{ann}} - P_{s_{cav}})}\right)d\hat{x}$$

$$= C_d A_{gap}\sqrt{2\rho}\int_{\sqrt{k}}^{1}\left(\sqrt{(P_{s_{ann}} - P_{s_{cav}})}\right)d\hat{x}$$

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Single-Orifice Model (9)

❖ Scanlon et al. (2004)

Mass Conservation at Rim Seal

❖ Ingress mass flow rate $\dot{m}_{ing}$

Substituting

$$P_{s_{ann}} - P_{s_{cav}} = (\hat{x}^2 - k)(P_{s_{max}} - P_{s_{min}})$$

yields

$$\dot{m}_{ing} = C_d A_{gap} \sqrt{2\rho(P_{s_{max}} - P_{s_{min}})} \int_{\sqrt{k}}^{1} \left(\sqrt{(\hat{x}^2 - k)} \right) d\hat{x}$$

$$\boxed{\dot{m}_{ing} = C_d A_{gap} \sqrt{2\rho(P_{s_{max}} - P_{s_{min}})} \left[\frac{1}{2} \left(\sqrt{1-k} - k \cosh^{-1} \frac{1}{\sqrt{k}} \right) \right]}$$

Single-Orifice Model (10)

❖ Scanlon et al. (2004)

❑ Mass Conservation at Rim Seal

- ❖ Egress mass flow rate $\dot{m}_{egr}$

$$\int d\dot{m}_{egr} = C_d\sqrt{2\rho}\int\left(\sqrt{(P_{s_{cav}} - P_{s_{ann}})}\right)dA_{gap} = NC_d\sqrt{2\rho}\int\left(\sqrt{(P_{s_{cav}} - P_{s_{ann}})}\right)d\hat{A}_{gap}$$

$$\dot{m}_{egr} = C_d A_{gap}\sqrt{2\rho(P_{s_{max}} - P_{s_{min}})}\int_0^{\sqrt{k}}\left(\sqrt{(k - \hat{x}^2)}\right)d\hat{x}$$

$$\boxed{\dot{m}_{egr} = C_d A_{gap}\sqrt{2\rho(P_{s_{max}} - P_{s_{min}})}\left(\frac{k\pi}{4}\right)}$$

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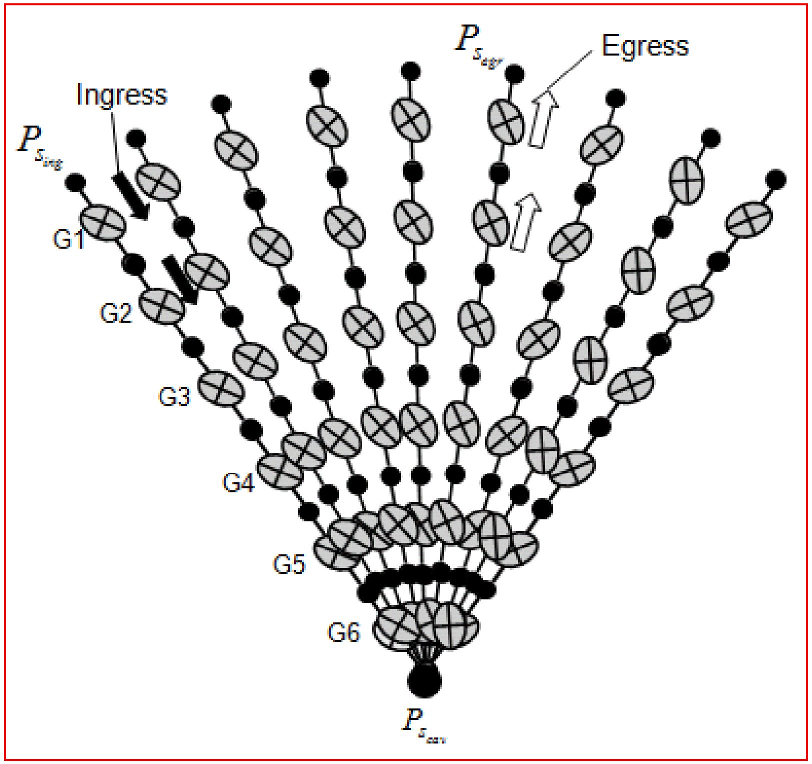
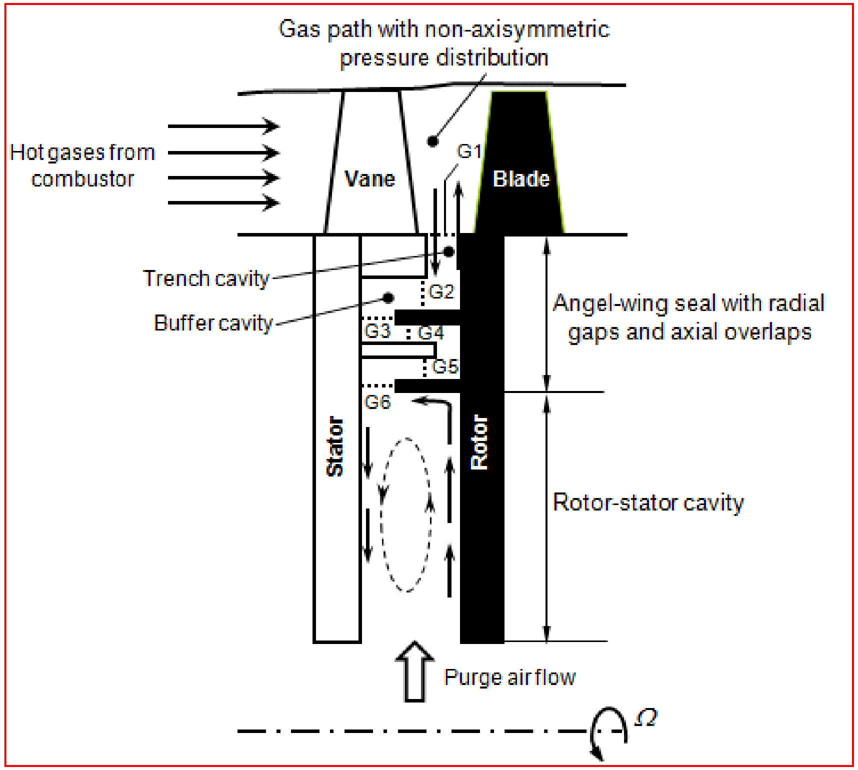
Multiple-Orifice Spoke Model (1)

Schematic of Multiple-Orifice Spoke Model

- Each spoke represents a serially-connected rim seal system of orifices.





Multiple-Orifice Spoke Model (2)

Mass Flow Rate through Each Orifice along a Spoke

- Any spoke-to-spoke interaction is neglected in this model.
- For a subsonic air flow, assumed here, the static pressure at the orifice exit equals the static pressure of the downstream node.

$$\boxed{\dot{m}_{ideal} = \rho A V_x} \qquad \boxed{\dot{m}_{real} = C_d \dot{m}_{ideal}}$$

- Method 1

Step 1. Calculate the static temperature: $T_s = T_t - V^2 / 2c_p$, where V is the total velocity at the orifice exit.

Step 2. Calculate the speed of sound: $C = \sqrt{\kappa R T_s}$

Multiple-Orifice Spoke Model (3)

- **Mass Flow Rate through Each Orifice along a Spoke**
  - **Method 1 (continued)**

Step 3. **Calculate the total-velocity Mach number:** $\mathbf{M = V / C}$

Step 4. **Calculate the static-pressure mass flow function:**

$$\boxed{\hat{F}_{f_s} = M\sqrt{\kappa\left(1+\frac{\kappa-1}{2}M^2\right)}}$$

Step 5. **Calculate the orifice ideal mass flow rate:**

$$\boxed{\dot{m}_{ideal} = \frac{A C_V \hat{F}_{f_s} P_s}{\sqrt{R T_t}}}$$

where the velocity coefficient $C_V = V_x / V$

Multiple-Orifice Spoke Model (4)

Mass Flow Rate through Each Orifice along a Spoke

Method 2

Step 1. Calculate the static temperature: $T_s = T_t - V^2 / 2c_p$, where V is the total velocity at the orifice exit.

Step 2. Calculate the speed of sound: $C = \sqrt{\kappa R T_s}$

Step 3. Calculate the axial-velocity Mach number: $M_x = V_x / C$

Step 4. Calculate the static-pressure mass flow function: $$\boxed{\hat{F}_{f_{s,x}} = M_x \sqrt{\kappa \left(1 + \frac{\kappa - 1}{2} M_x^2\right)}}$$

Step 5. Calculate the orifice ideal mass flow rate: $$\boxed{\dot{m}_{ideal} = \frac{A \hat{F}_{f_{s,x}} P_s}{\sqrt{R T_{t_x}}}}$$

where $T_{t_x} = T_s + V_x^2 / 2c_p$

Recommended Design Philosophy to Handle Hot Gas Ingestion with Minimum Performance Penalty

- Establish the minimum cavity purge flow needed for an acceptable windage temperature rise and heat transfer in the rotor-stator cavity.
- Establish gas path **asymmetric** pressure boundary conditions from appropriate CFD solution.
- Design a seal that will limit ingress (hot gas ingestion) to trench (**first design target!**) and buffer cavities (**second design target if we can't meet the first!!**).

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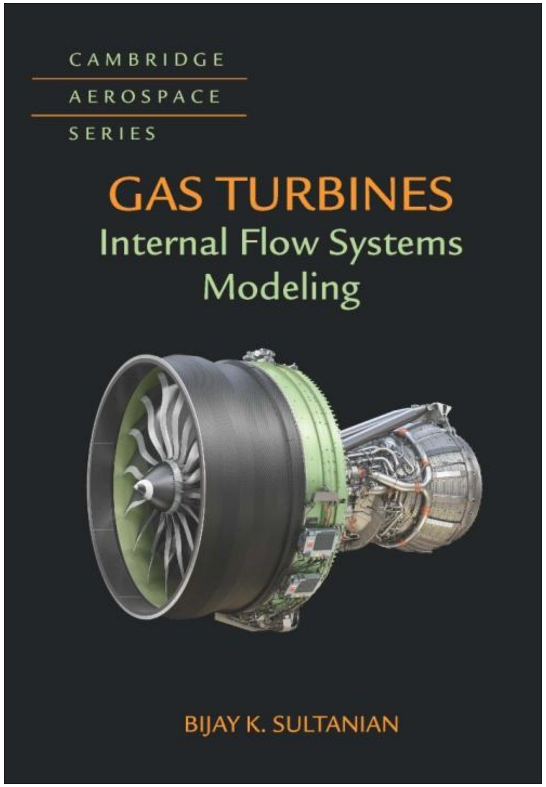
QUESTIONS?

THANK YOU!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Physics-Based Modeling of Gas Turbine Secondary Air Systems

Module 6: Physics-Based Modeling – Part III

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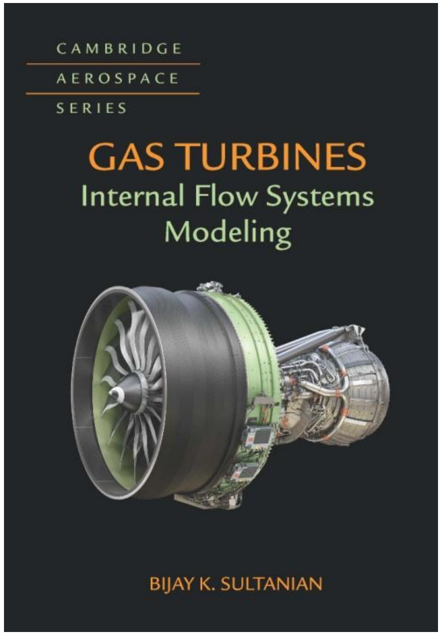

ASME 2019 TURBO EXPO
Phoenix, Arizona, USA
Sunday, June 16, 2019

Module 6

Physics-Based Modeling – Part III

Module 6

Physics-Based Modeling – Part III

- Whole engine modeling (WEM)
- Network of convection links
- Multisurface forced vortex convection link with windage
- Junction treatment in the network of convection links
- Validation with Engine Test Data
- Key recommendations on SAS modeling

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Whole Engine Modeling (1)

Key Goals

- Durability considerations of critical gas turbine components, which are life-limited because of creep, oxidation, low cycle fatigue (LCF), and high cycle fatigue (HCF)
- Life management of critical components through scheduled maintenance, refurbishment, and replacement
- Multiphysics (aero-thermal-mechanical)-based whole engine modeling (WEM) is foundational to Internet of Things (IoT) revolution and for developing the engine digital twin, leveraging actual service data in the predictive models.
- The transient multiphysics WEM is needed for controlling clearances at the blade tips in the gas path and also for seals in the secondary air systems.

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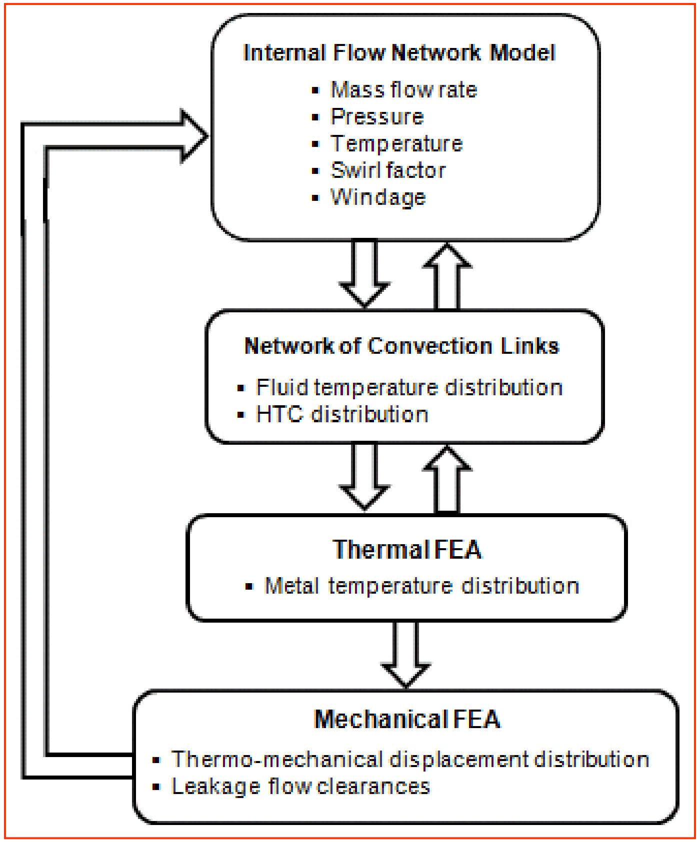
Whole Engine Modeling (2)

Multiphysics Modeling of Engine Transients

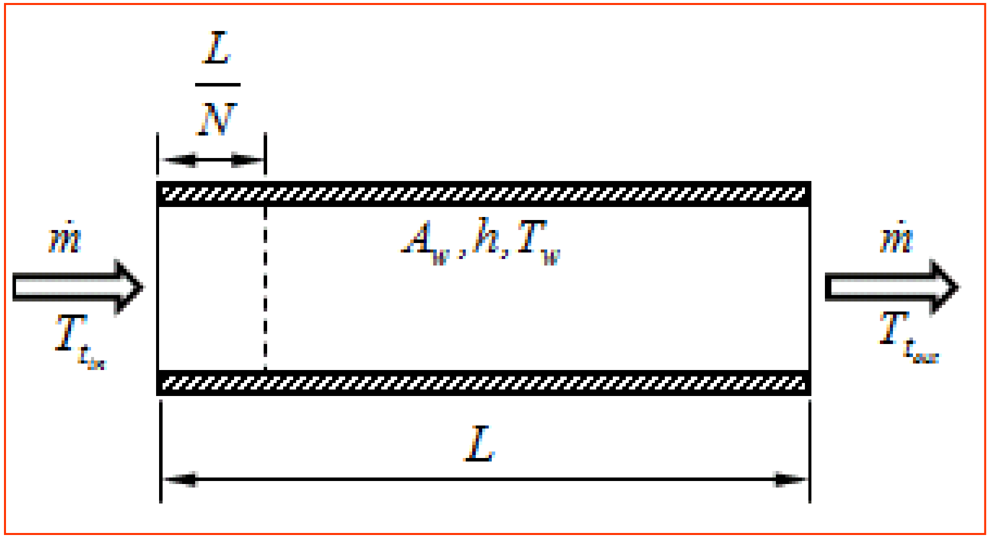
Layered flow network modeling methodology

- The thermal analysis problem in gas turbines uniquely involves convective boundary conditions that themselves depend on the thermal solution elsewhere and cannot be specified a priori.
- The gas turbine internal flow systems respond orders of magnitude faster (convection time constant) than the thermal response (diffusion time constant) of the structural members in contact.



Network of Convection Links (1)

Linear Convection Link



$$\dot{Q}_c = hA_w\left(T_w - \frac{T_{t_{in}} + T_{t_{out}}}{2}\right) = \frac{hA_w}{2}\left\{(T_w - T_{t_{in}}) + (T_w - T_{t_{out}})\right\}$$

$$\dot{m}c_p(T_{t_{out}} - T_{t_{in}}) = \dot{Q}_c = \frac{hA_w}{2}\left\{(T_w - T_{t_{in}}) + (T_w - T_{t_{out}})\right\}$$

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Network of Convection Links (2)

Linear Convection Link

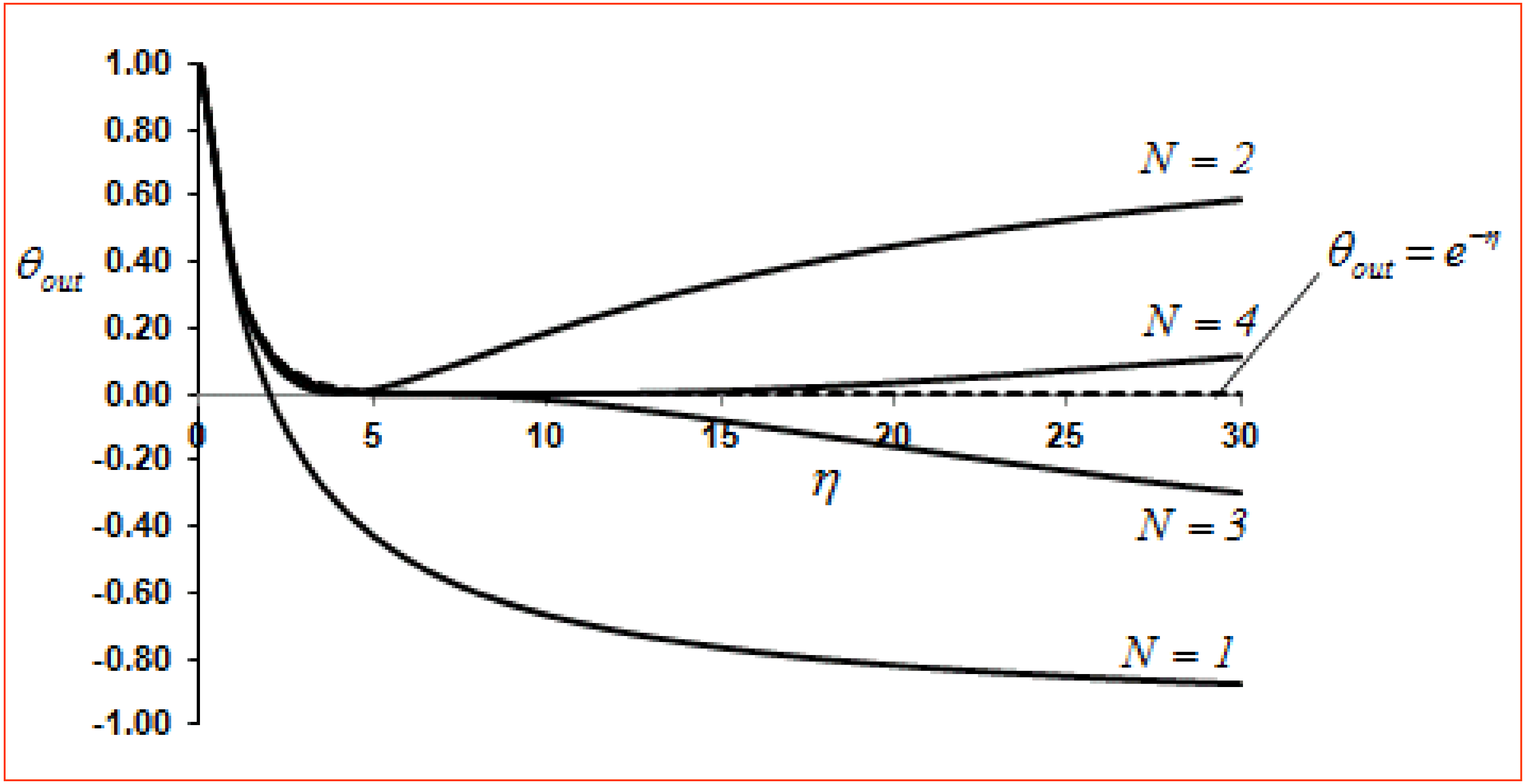
$$\boxed{\left\{(T_w - T_{t_{in}}) - (T_w - T_{t_{out}})\right\} = \frac{\eta}{2}\left\{(T_w - T_{t_{in}}) + (T_w - T_{t_{out}})\right\}}$$

$$\boxed{1 - \theta_{out} = \frac{\eta}{2}(1 + \theta_{out})} \qquad \boxed{\theta_{out} = \frac{2-\eta}{2+\eta}} \qquad \boxed{\eta = \frac{hA_w}{\dot{m}c_p}} \qquad \boxed{\theta_{out} = \frac{T_w - T_{t_{out}}}{T_w - T_{t_{in}}}}$$

$$\boxed{\theta_{out} = \left(\frac{2 - \dfrac{\eta}{N}}{2 + \dfrac{\eta}{N}}\right)^N}$$

Network of Convection Links (3)

❑ Linear Convection Link



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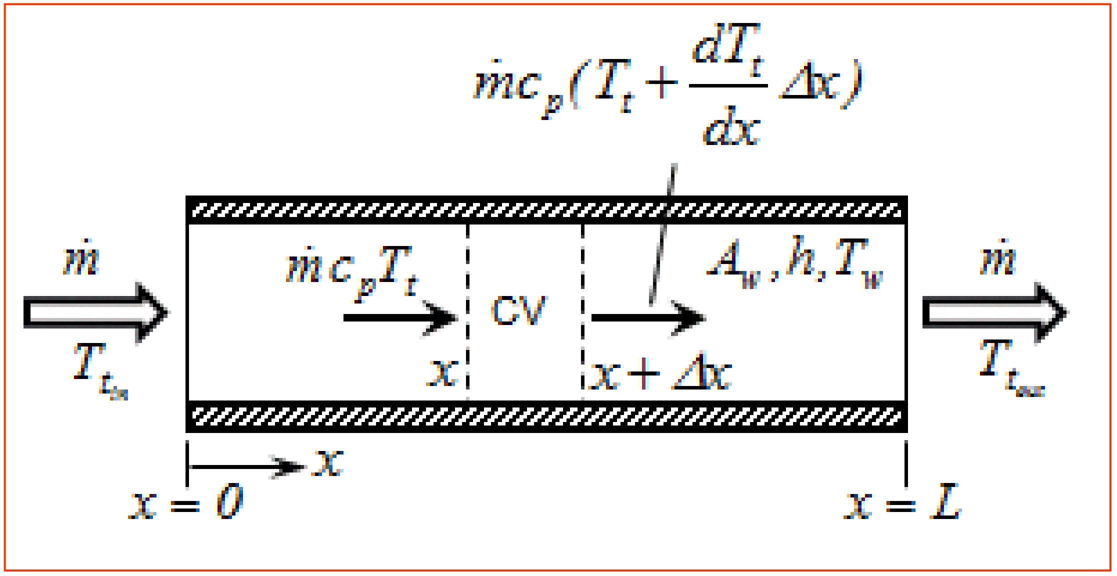
Network of Convection Links (4)

❑ Nonlinear Convection Link



$$\dot{m}c_p\left(T_t + \frac{dT_t}{dx}\Delta x\right) - \dot{m}c_pT_t = h\frac{A_w}{L}(T_w - T_t)\Delta x$$

Network of Convection Links (5)

❑ Nonlinear Convection Link

$$\boxed{\frac{dT_t}{d\xi} = \eta(T_w - T_t)} \qquad \boxed{\xi = \frac{x}{L}} \qquad \boxed{\theta = \frac{T_w - T_t}{T_w - T_{t_{in}}}}$$

$$\boxed{\frac{d\theta}{d\xi} + \eta\theta = 0}$$

➢ With the boundary condition: $\theta = 1$ at $\xi = 0$, we obtain the solution

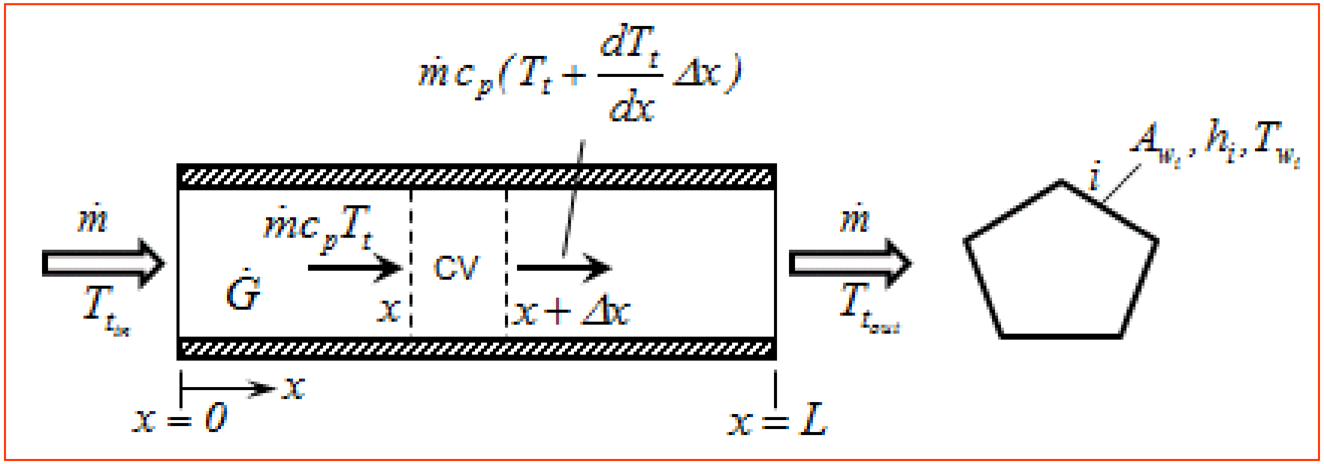
$$\boxed{\theta = e^{-\eta\xi}}$$

➢ At outlet ($\xi = 1$): $\boxed{\theta_{out} = e^{-\eta}}$ for $0 \le \eta \le \infty$

Network of Convection Links (6)

Nonlinear Convection link in a Multisided Duct

Without Internal Heat Generation



$$\dot{m}c_p\left(T_t + \frac{dT_t}{dx}\Delta x\right) - \dot{m}c_p T_t = \sum_{i=1}^{i=N} h_i \frac{A_{w_i}}{L}(T_{w_i} - T_t)\Delta x$$

$$\dot{m}c_p \frac{dT_t}{d\xi} = \sum_{i=1}^{i=N} h_i A_{w_i} T_{w_i} - \sum_{i=1}^{i=N} h_i A_{w_i} T_t$$

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Network of Convection Links (7)

Nonlinear Convection link in a Multisided Duct

Without Internal Heat Generation

$$\boxed{\frac{d\theta}{d\xi} + \bar{\eta}\theta}$$

$$\boxed{\bar{\eta} = \frac{\sum_{i=1}^{i=N} h_i A_{w_i}}{\dot{m} c_p}}$$

$$\boxed{\theta = \frac{\bar{T}_w - T_t}{\bar{T}_w - T_{t_{in}}}}$$

$$\boxed{\bar{T}_w = \frac{\sum_{i=1}^{i=N} h_i A_{w_i} T_{w_i}}{\sum_{i=1}^{i=N} h_i A_{w_i}}}$$

- With the boundary condition: $\theta = 1$ at $\xi = 0$, we obtain the solution

$$\boxed{\theta = e^{-\bar{\eta}\xi}}$$

- At outlet ($\xi = 1$): $\boxed{\theta_{out} = e^{-\bar{\eta}}}$ for $0 \le \bar{\eta} \le \infty$

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Network of Convection Links (8)

- Nonlinear Convection link in a Multisided Duct
  - With Uniform Internal Heat Generation

$$\dot{m}c_p\left(T_t + \frac{dT_t}{dx}\Delta x\right) - \dot{m}c_p T_t = \sum_{i=1}^{i=N} h_i \frac{A_{w_i}}{L}(T_{w_i} - T_t)\Delta x + \dot{G}\frac{\Delta x}{L}$$

$$\dot{m}c_p\frac{dT_t}{d\xi} = \sum_{i=1}^{i=N} h_i A_{w_i} T_{w_i} - \sum_{i=1}^{i=N} h_i A_{w_i} T_t + \dot{G}$$

$$\frac{d\theta}{d\xi} + \bar{\eta}\theta = -\tilde{\varepsilon}$$

$$\tilde{\varepsilon} = \frac{\dot{G}}{\dot{m}c_p(T_w - T_{t_{in}})}$$

Network of Convection Links (9)

Nonlinear Convection link in a Multisided Duct

With Uniform Internal Heat Generation

- **Solution:**

$$\boxed{\theta = C\theta_h + \theta_P} \qquad \boxed{\theta_h = e^{-\bar{\eta}\xi}} \qquad \boxed{\theta_P = \tilde{\varepsilon}/\bar{\eta}}$$

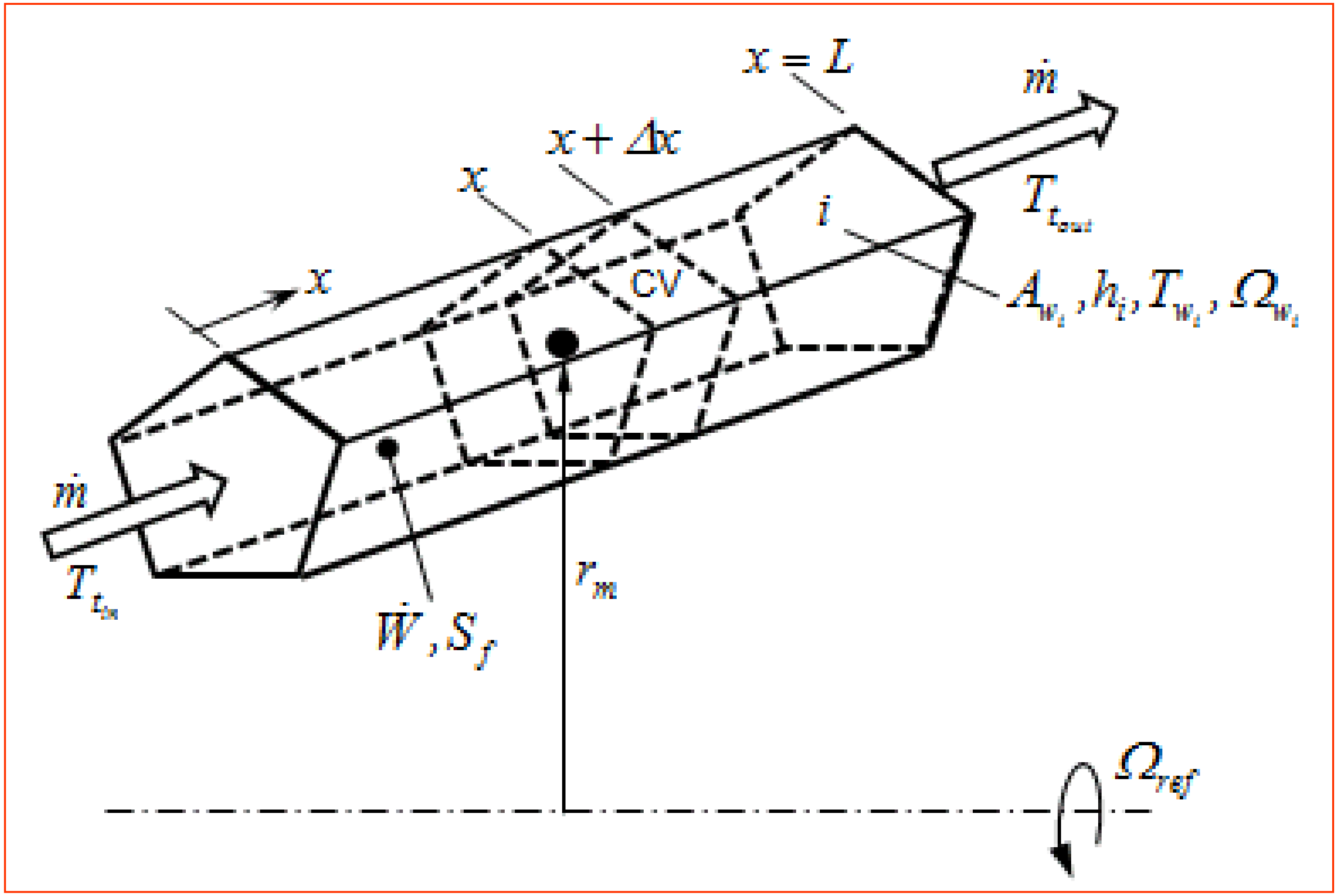
- **With the boundary condition:** $\theta = 1$ **at** $\xi = 0$

$$\boxed{\theta = \left(1 + \frac{\tilde{\varepsilon}}{\bar{\eta}}\right) e^{-\bar{\eta}\xi} - \frac{\tilde{\varepsilon}}{\bar{\eta}}}$$

- **At outlet (** $\xi = 1$ **):** $\boxed{\theta_{out} = \left(1 + \frac{\tilde{\varepsilon}}{\bar{\eta}}\right) e^{-\bar{\eta}} - \frac{\tilde{\varepsilon}}{\bar{\eta}}}$ **for** $0 \le \bar{\eta} \le \infty$

Network of Convection Links (10)

- Multisurface Forced Vortex Convection Link with Windage



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Network of Convection Links (11)

Multisurface Forced Vortex Convection Link with Windage

$$\boxed{\dot{m}c_p\left(T_t+\frac{dT_t}{dx}\Delta x\right)-\dot{m}c_pT_t=\sum_{i=1}^{i=N}h_i\frac{A_{w_i}}{L}(T_{w_i}-T_t)\Delta x+\dot{W}\frac{\Delta x}{L}}$$

$$\boxed{\frac{dT_t}{d\xi}=\bar{\eta}\bar{T}_w-\sum_{i=1}^{i=N}\frac{h_iA_{w_i}T_{aw_i}}{\dot{m}c_p}+\frac{\dot{W}}{\dot{m}c_p}}$$

where

$$\boxed{T_{aw_i}=T_t-\frac{r_m^2S_f^2\Omega_{ref}^2}{2c_p}+\frac{r_m^2\,Pr^{1/3}}{2c_p}\left(S_f\Omega_{ref}-\Omega_{w_i}\right)^2}$$

$$\boxed{T_{aw_i}=T_t-\frac{r_m^2S_f^2\Omega_{ref}^2}{2c_p}+\frac{r_m^2\,Pr^{1/3}}{2c_p}\left(S_f^2\Omega_{ref}^2-2S_f\Omega_{ref}\Omega_{w_i}+\Omega_{w_i}^2\right)}$$

$$\boxed{T_{aw_i}=T_t-\frac{r_m^2(1-Pr^{1/3})S_f^2\Omega_{ref}^2}{2c_p}-\frac{r_m^2\,Pr^{1/3}\,S_f\Omega_{ref}\Omega_{w_i}}{c_p}+\frac{r_m^2\,Pr^{1/3}\,\Omega_{w_i}^2}{2c_p}}$$

Network of Convection Links (12)

❑ Multisurface Forced Vortex Convection Link with Windage

Substituting T_{aw_i} in the ODE yields

$$\boxed{\frac{d\theta}{d\xi} + \overline{\eta}\theta = -\hat{\varepsilon}}$$

where

$$\boxed{\hat{\varepsilon} = \frac{\overline{\eta} r_m^2 (1 - Pr^{1/3}) S_f^2 \Omega_{ref}^2}{2c_p(\overline{T}_w - T_{t_{in}})} + \frac{r_m^2\, Pr^{1/3}\, S_f \Omega_{ref} \overline{\eta} \overline{\Omega_w}}{c_p(\overline{T}_w - T_{t_{in}})} - \frac{r_m^2\, Pr^{1/3}\, \overline{\eta} \overline{\Omega_w^2}}{2c_p(\overline{T}_w - T_{t_{in}})} + \frac{\dot{W}}{\dot{m} c_p(\overline{T}_w - T_{t_{in}})}}$$

$$\boxed{\overline{\Omega_w} = \frac{\sum_{i=1}^{i=N} h_i A_{w_i} \Omega_{w_i}}{\sum_{i=1}^{i=N} h_i A_{w_i}}}$$

$$\boxed{\overline{\Omega_w^2} = \frac{\sum_{i=1}^{i=N} h_i A_{w_i} \Omega_{w_i}^2}{\sum_{i=1}^{i=N} h_i A_{w_i}}}$$

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Network of Convection Links (13)

Multisurface Forced Vortex Convection Link with Windage

Thus, with the boundary condition: $\theta = 1$ **at** $\xi = 0$

$$\boxed{\theta = \left(1 + \frac{\hat{\varepsilon}}{\bar{\eta}}\right) e^{-\bar{\eta}\xi} - \frac{\hat{\varepsilon}}{\bar{\eta}}}$$

➢ **At outlet (** $\xi = 1$ **):** $\boxed{\theta_{out} = \left(1 + \frac{\hat{\varepsilon}}{\bar{\eta}}\right) e^{-\bar{\eta}} - \frac{\hat{\varepsilon}}{\bar{\eta}}}$

$$\boxed{T_{t_{out}} = \bar{T}_w - (\bar{T}_w - T_{t_{in}}) e^{-\bar{\eta}} + (1 - e^{-\bar{\eta}}) \tilde{E}}$$

where

$$\boxed{\tilde{E} = \frac{r_m^2}{2c_p}\left[(1 - Pr^{1/3}) S_f^2 \Omega_{ref}^2 + 2 Pr^{1/3} S_f \Omega_{ref} \overline{\Omega_w} - Pr^{1/3} \overline{\Omega_w^2}\right] + \frac{\dot{W}}{\sum_{i=1}^{i=N} h_i A_{w_i}}}$$

Network of Convection Links (14)

Junction Treatment

- **Mixed mean total temperature of all convection links flowing into junction J:**

$$T_{t_J} = \frac{\sum_{i=1}^{i=k} \dot{m}_i T_{t_{out_i}}}{\sum_{i=1}^{i=k} \dot{m}}$$

- **Mixed mean fractional swirl velocity of all convection links flowing into junction J:**

$$S_{f_J} = \frac{\sum_{i=1}^{i=k} \dot{m}_i S_{f_i}}{\sum_{i=1}^{i=k} \dot{m}}$$

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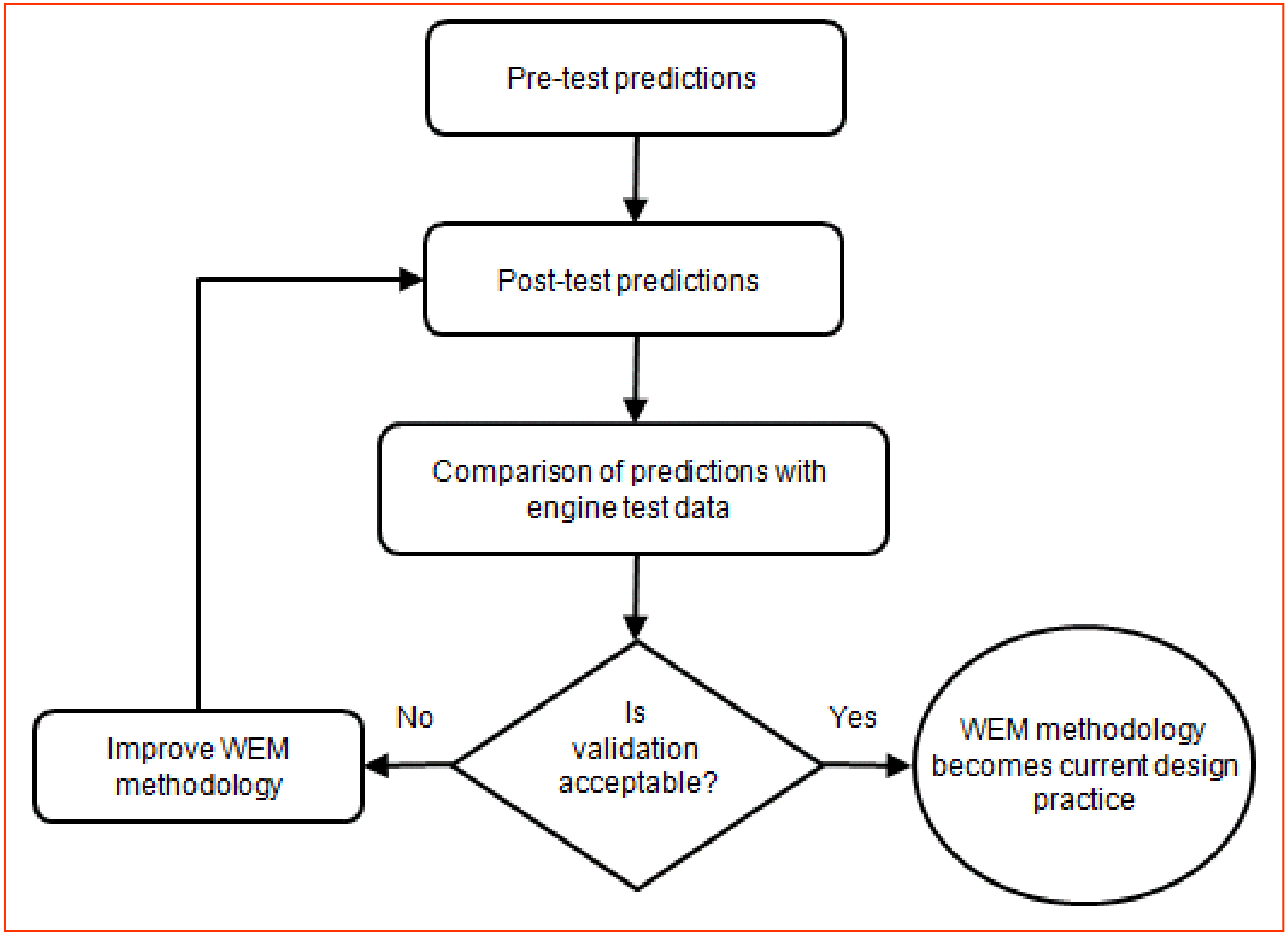
Network of Convection Links (15)

Junction Treatment

- For a downstream forced vortex with the specified swirl factor S_{f_j}

$$\boxed{T_{t_{j_{in}}} = T_{t_J} + \frac{r_j^2 \Omega_{ref}^2 (S_{f_j} - S_{f_J}) S_{f_j}}{c_p}}$$

Validation with Engine Test Data



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Key Recommendations for Improved SAS Modeling (1)

- For accurate computation of pressure and temperature distributions in a **rotor-stator cavity**, model them as a "**parabolic generalized vortex,**" satisfying conservation equations in each sub-cavity.
- Use a static-pressure-based formulation to model a duct component with **area change**, **friction**, and **heat transfer coupled with rotational pumping**. This will establish **synergy/commonality** with the flow network modeling of internal cooling of airfoils.
- Use multiple-orifice network model as a **super-component** to handle the gas path sealing design to prevent **hot gas ingestion**.

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Key Recommendations for Improved SAS Modeling (2)

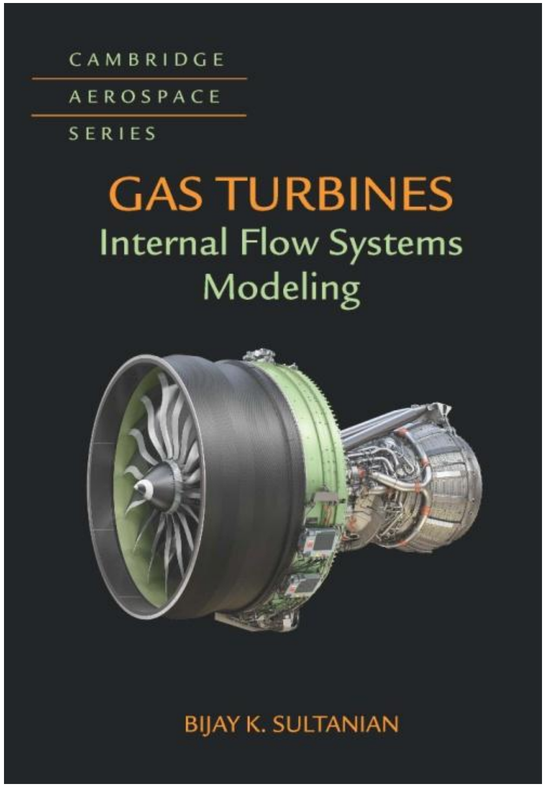
- **Use 3D CFD modeling to better understand the flow physics and to reinforce modeling assumptions of SAS components, where needed.**

- **Include accurate modeling of heat transfer in SAS flow network using a network of convection links interfacing the metal temperature prediction method.**

- **In view of uncertainties associated with various input and boundary condition data, it behooves us to perform a probabilistic SAS model analysis to ensure robustness of critical response quantities, e.g., turbine airfoil cooling flow rate!**

QUESTIONS?

THANK YOU!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